

Effects of Ad-Blockers Adoption on Digital Piracy: A Blessing or a Curse?

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Abstract

This paper studies the effect of ad-blockers adoption on digital piracy in a duopoly market with a legal platform offering video-on-demand services, and a pirate platform relying on advertisement revenues. We show that the effect of ad-blockers adoption on the market structure is sensitive to the type of welfare maximisation problem pursued by the ad-blocker. Our results demonstrate that by partially reducing the level of advertisements, the ad-blocker adoption exerts competitive pressure on the legal firm, shifting some of its pre-adoption demand in favour of the pirate demand (a pro-piracy effect) prompting the legal firm to lower its prices to recover some of its (lost) demand. We further show that when the ad-blocker pursues an unconstrained consumer' surplus maximisation problem, it can either induce a zero level of advertisement or an excessively high level of advertisement both of which induce consumers to use only the legal platform. In this case, the presence of ad-blockers grants a monopoly position to the legal firm. We further discuss various policy implications emerging from our analysis.

JEL Classification: L11, L86, M37.

Keywords: Digital Piracy, Advertisement, Platform competition, Ad-blockers.

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1 Introduction

The growing digitalisation of contents has been historically pervaded by the presence of illicit behaviours. In the past decade, software producers and blockbusters were mainly concerned about the availability of pirate copies at low or even nil prices due to low reproduction costs of pirate copies. This has stimulated extensive research about effective ways to deter and fight user-piracy (Belleflamme 2003; Banerjee 2003, amongst others). Whilst copied CDs had been the main driver of piracy in the past decade¹, online piracy has become more common in recent years with users sharing their files and cracked versions of copyrighted software, motivated by altruistic reasons due to the diffusion of peer-to-peer networks in recent years².

Whilst these forms of piracy still exist and have been extensively analysed in the economic literature³, nowadays, a new form of copyright infringement and digital piracy has emerged due to the widespread prevalence of various platforms and media-players (of mainly commercial nature) enabling users to watch movies, shows or series directly online, that directly challenges the established video-on-demand platforms such as Netflix, Amazon amongst others. A leading example has been the case of Megavideo, one of the services offered by Kim Dotcom and its company Megaupload Ltd., before its shut down ordered by the US Department of Justice in 2012.⁴ The company transformed the not-for-profit goal of file-sharing platforms into a sponsored business model, allowing for both downloading (through Megaupload) and streaming of video contents (through Megavideo) in exchange of advertisements. Similarly and more recently, The Pirate Bay, another website providing reproduction and distribution of copyrighted video contents, made approximately 24.5 million of USD dollars as annual advertisements revenues (reported in Chang and Walter 2015).

Despite an increasing number of court-ordered seizures, similar platforms continue to operate⁵ and the extent of the competition between them and legal platforms has be-

¹Before digitalisation, the study of piracy and copying of informational goods characterized by non-excludability and non-rivalry was concerned about unauthorised copied materials and intellectual property protection (see Johnson 1985; Liebowitz 1985).

²See Casadesus-Masanell and Hervas-Drane (2010a; 2010b) and Klumpp (2014) for the analysis of the evolution of peer-to-peer networks.

³For a survey, see Peitz and Waelbroeck (2006b); Belleflamme and Peitz (2010; 2014); Gomes et al. (2015). For a critical overview of recent developments, see Belleflamme (2016).

⁴*United States vs. Kim Dotcom. Indictment, No. 1:12CR3* (US District Court for the Eastern District of Virginia 2012). <http://www.wsj.com/articles/SB10001424052970204616504577171180266957116>. Last visit on 15 October 2016.

⁵Nowvideo, a hosting platform similar to Megavideo, has recently drawn the attention of the Italian

come so relevant that, in 2015, Netflix declared to its shareholder that "piracy continues to be one of our biggest competitors"⁶.

To fight piracy, one of the strategies adopted by video- and music-content streaming companies has been to segment their markets and apply discriminatory pricing: e.g. Spotify started offering a "Freemium" model, that is, combining free music in presence of advertisements and restrictions on mobile phones with ads-free accounts at a monthly price; Netflix promoted one-month free-trial to enable users to learn about its platform, also offering "Family" packages and multiple accounts. These strategies have demonstrated their effectiveness to fight user piracy, with declining trends in many countries.⁷ However, the magnitude of the traffic on commercially-oriented platforms hosting free copyrighted video-contents and monetising users visits through cross-side externalities, still remains to be estimated.⁸ Using the jargon, these platforms are called cyber-lockers, or one-click hostings - they do not directly upload materials violating copyrights, but rely on users uploading and on the traffic generated by third-party websites sponsoring the links. As the product no longer needs to be downloaded and can be watched on-line being accessible via any device, the competition between hosting platforms (that we will call PP, pirate platform) and legal platforms (LP) becomes fiercer as the product (same content but provided by different platforms) gets less differentiated⁹.

Advertisements displayed on pirate platforms are usually very invasive, often hiding malwares or leading to malicious websites, and can be interpreted as a non-monetary price charged to users.¹⁰ Consumers on the other hand can protect themselves from such "nuisance" advertising and behavioural profiling by installing plug-ins in their browsers, known as ad-blockers, that can potentially block such unwanted advertisements. One

Communications Authority (Agcom: DDA/0000551), after a request made by Sky Italia, an Italian pay-per-view, about copyright infringements related to several movies (e.g. Olive Kitteridge - Season 1, The Borgias - Season 3 and many others) and shows (e.g. Masterchef - Season 4; X-Factor - Season 8, etc.), whose episodes were freely available on their network.

⁶The 2015 Netflix letter to shareholders, p. 5.

⁷According to Sandvine (2011), Netflix overtook BitTorrent in providing video content in 2011.

⁸See the literature on two-sided markets: Rochet and Tirole (2003; 2006), Armstrong (2006), amongst others. For a not-exhaustive survey, see Rysman (2009) and Zingal and Becker (2013).

⁹Despite the presence of different legal video-on-demand platforms, each of them has a sort of monopolistic position as offering different catalogues of products. Instead, for a single product, the competition is only between a pirate platform and the legal platform, owner of temporary or exclusive property rights

¹⁰Rafique et al. (2016) reported that half of free live streaming websites is "malicious in nature, offering malware (zero-day in one case), showing fake law enforcement messages to collect purported fines, and luring users to install malicious browser extensions" (p. 2)

famous example of such an ad-blocker is Ad-Block Plus. The adoption of ad-blockers has indeed gone through a rapid growth in Western countries in recent years, e.g. the use of ad-blockers has gone up by as much as 41% between 2014 and 2015. Inevitably, the adoption of ad-blockers poses a threat for the survival of sponsored-business models as it implies a loss of revenue for such firms. For example, the use of ad-blockers is expected to lead to a revenue loss of 20 billion of USD in 2016¹¹.

Motivated by above facts, in this paper we analyse the price-advertisement competition between legal and pirate firms in the video-on-demand market to examine the impact of ad-blocker adoption on digital piracy. This is relevant because ad-blocker adoption reduces the advertisement flow on the pirate platform which, while hurts the pirate firm's revenues, can be potentially beneficial for consumers as ad-blocking reduces the annoyance cost of advertisements. To the best of our knowledge, ours is the first study on the impact of ad-blockers adoption on the competition between the legal and pirate platforms. Whilst there exist a small number of papers that directly study the competition between legal and pirate firms (Chang and Walter 2015), the most of the literature in that field focuses only on deterrence strategies. Instead, the role of ad-blockers have been investigated mainly for its consequences for legally established platforms (see Anderson and Gans (2011)).

We analyse a theoretical model of platform competition with vertical and horizontal product differentiation with two types of markets: a pre-adoption market where a pure price-advertisement competition takes place between the legal and pirate firm; and the post-adoption market where the legal firm sets its price and the ad-blocker (replacing the pirate firm, which then simply becomes passive) decides on the amount of advertisements to be allowed to consumers on the pirate screen. The ad-blocker in our model (which is not a pure social welfare maximiser) maximizes the consumer surplus, under the condition that profits of the pirate firm be non-negative. For each of the above scenarios, we consider two cases where (i) the legal firm leads the game moving as a Stackelberg leader; (ii) the pirate firm (or the ad-blocker substituting for the pirate firm) leads the game, by deciding first on the amount of advertisements.

We show that, regardless of which firm moves first, advertisements and prices are strategic complements. As a consequence, the adoption of ad-blockers exerts a competi-

¹¹This plug-in has become very popular in recent years, reaching around 12 million of monthly users in the UK in 2015, 77 million in Europe with rate of adoption of 35% in Poland and 27% in Greece See Fairpage, Report on the cost of ad-blocking available on https://downloads.pagefair.com/wp-content/uploads/2016/05/2015_report-the_cost_of_ad_blocking.pdf. Last visit on 15 October 2016.

tive pressure on the legal firm, shifting some of its pre-adoption demand in favour of the pirate demand (a pro-piracy effect) prompting the legal firm to lower its prices in order to recover some its (lost) demand (a strategic effect). However, the strategic effect is not strong enough to fully compensate the direct pro-piracy effect. Consequently, the legal firm is adversely affected by the presence of ad-blockers! A curse for the LP!

We further extend our analysis to consider a purely benevolent ad-blocker who is concerned *only* about consumers' welfare and hence maximises consumers' surplus regardless of its effects on the pirate firm's profit. We show that, interestingly, there can arise two contrasting possible equilibrium scenarios in this case. In the first scenario, the ad-blocker fully blocks all the advertisements which implies that in the presence of fixed costs, the pirate sponsored business is no longer sustainable: this puts the legal firm in a monopoly position by default. In the second scenario, the adoption of ad-blockers actually enhances the level of advertisements relative to the pre-adoption case to such an extent that rational consumers neglect to install the blocking software! A blessing for the LP!

The rest of the paper is as follows. In Section 2, we provide a review of previous analysis and conclusions concerning deterrence and accommodation of piracy. In Section 3, we introduce the model. In Section 4 we consider a sequential-move game with the LP moving first, followed by advertisement decision first set by the pirate firm, and then by the ad-blocker. Section 5 presents the model with the LP moving as a follower. Section 6 concludes the paper.

2 Related Literature

Our paper relates to several streams of the literature. In particular, it relates to the analysis of piracy, informational goods and digital markets, and on the strategies aimed to deter or accommodate illicit behaviours. Belleflamme (2003) defined a model with a monopolist facing the decision about fully blocking, deterring or accommodating users piracy. He showed that, in the short-run, piracy, when not fully blocked, contributes to increase the social welfare by reducing the market power of the monopolist. Yoon (2002) provided a welfare analysis showing that copyright protection in the presence of a monopolist can either increase or decrease the social welfare depending on problems of underproduction or underutilisation. He also showed that the optimal level of copyright production has an all-or-nothing characteristic that does not allow outcomes at

intermediate levels.

Bae and Choi (2006) found that piracy lowers the price for legal softwares, actually inducing more demand. They also found that a crucial role is played by two types of costs often associated to piracy: the reproduction cost of copying and the degradation cost associated to the quality of the pirate copy. Belleflamme and Picard (2006) studied piracy in a framework characterised by a multi-product firm in both monopoly and duopoly setting. Herings et al. (2010) considered the case of a physical product competing against a digitalised version available in a peer-to-peer network, in presence of taste heterogeneity, network effects and endogenous pricing strategies. They showed that, in the presence of relatively larger downloading costs, there is a lower probability of formation of the peer-to-peer network and, indeed, monopolistic outcomes are more likely, whereas profits are negatively affected for reduction in these costs. Minniti and Vergari (2010) studied piracy arising from peer-to-peer networks, showing that the extent of the market coverage is relevant in determining the effect of piracy. They showed that only in a market with partial coverage consumers benefit from piracy as a consequence of demand effects, whilst under full coverage, piracy represents a problem which can erode market shares. Ahn and Shin (2010) incorporated digital right managements (DRM) as alternative to copyright protection to fight piracy. Inceoglu (2015) showed that piracy can benefit the incumbent, preventing the entry of other legal competitors, but still benefitting consumers as providing a low-cost alternative to the pricey legal good.

On a somewhat different note, using a two-sided model with piracy, Rasch and Wenzel (2013; 2015) studied the effect of piracy in a vertically differentiated market with illegal and legal software, developers and consumers. In the first paper, they found that the effect of piracy on profits and prices can be ambiguous depending upon whether platforms are compatible or not. When these are incompatible, it is shown that both the software developers and platforms are aligned in their interest and care about piracy. This is not the case in presence of incompatible platforms, where developers charge more in presence of higher software protection and the effect is not unidirectional for consumers. Other papers, instead, focused on the role of governments in sustaining anti-piracy policies (Banerijee 2003; Martinez-Sanchez 2010; Lopez-Cunat and Martinez-Sanchez 2010).

Our idea complements the paper of Chang and Walter (2015), which is also the only paper concerning price competition and advertisements. Their work, however, focussed on asymmetric competition where the legal firm selects the optimal price,

whereas the peer-to-peer hosting network optimizes choosing the quality of the good. They differentiated their findings according to whether the market is fully or partially covered, demonstrating that in the latter case the optimal investment in quality of the peer-to-peer network is higher than in the case with full market coverage. However, they relied on P2P networks, where the availability and the related downloading costs depends on the number of users connected to the platform. In our case, instead, we consider an illegal streaming service, whose quality and buffer are exogenous to the number of users. Additionally, as in Chang and Walter (2015), we assume that the revenue per visit is exogenous to the streaming service but the amount of advertisement displayed in each web-page is a strategic decision of the pirate company.

There are other studies based on different forms of competition that are close to ours that challenge the traditional business model based on one-sided interaction between buyers and sellers (Casadesus-Masanell and Zhu 2010; 2013). Casadesus-Masanell and Zhu (2010) analysed four different business models ranging from the subscription-based to the ad-sponsored and from a mixed to dual models. They showed that faced with the threat of ad-sponsored entrants, incumbent firms should prefer business models based on pure subscription or advertising revenues, not relying, indeed, on hybrid models that are more exposed to the risk of market cannibalisation.

By examining competition between on-line streaming platforms, our work is related to the stream of theoretical works on the music industry that can extended to analyse the functioning of video-on-demand (VOD) platforms. Halmenschlager and Waelbrock (2014) studied whether and to what extent online free music streaming services (e.g. Spotify, Deezer, etc.) can represent an effective way to fight piracy. They found that strategies as "Freemium" based on pricing and advertising discrimination can help to fight digital piracy and work as substitutes for strong copyright protections as long as the number of restrictions in the free case is limited. The "Freemium" model is also studied by Thomes (2013), who found that when a monopolist, providing both sponsored-supported version and a Premium version at a flat rate, faces a pirate peer-to-peer as competitor, it chooses the highest possible quality for the free service so as to completely offset the demand for illicitly copyrighted products.

Still on the music industry, several papers investigated the effect of free streaming (legal and pirate) on the music industry. Gayer and Shy (2006) and Piolatto and Schuett (2012) provided a theoretical appraisal of the effect of piracy on musicians, whose product can be available in the legal market, or copied, and whose revenues are dependent on legal copies and live performances. In this regard, Piolatto and Schuett (2012) found

that piracy worsens musicians' conditions according to their popularity and the relative importance of side revenues. Gans (2015) looked at the trade-off between fame and fortune. Empirical works offered controversial results. For instance, Mortimer et al. (2012) found that free availability on peer-to-peer networks displace sales of CDs but increase the demand for concerts with differences according to the popularity of the artists. Similar results on concerts were previously found by Oberholzer-Gee and Strumpf (2007). Likewise, Aguiar and Martens (2016) showed that free music streaming complements and not substitute legal downloading. For Bacache-Beauvallet et al. (2015) effects on artists depend on how frequently they are on-stage, with non-uniform results. It is also found that those artists frequently on-stage or without any contract are more tolerant of piracy, compared to those artists with little activity on-stage. Lee (2016) demonstrated that file sharing activities significantly affect music sales but the effect is quite small, with results depending on the fame of the artists.

Few studies are explicitly related to the growing sector of video-on-demand, which is the focus of our analysis. Among these, Danaher et al. (2010) used a quasi-experiment to assess the effect of the NBC's decisions to remove its content from iTunes on the illegal downloading of NBC's contents through BitTorrent. The estimated effect is quite enough, accounting for an increase of piracy of 11.4%. Peukert et al. (2013) studied the shut-down of Megaupload, finding that it negatively affected box offices revenues, but only for smaller and mid-range movies. Still on Megaupload, Danaher and Smith (2014) studied the effect of the US court decision to seize the popular content-provider on sales and digital, demonstrating an increase of the latter of 6.5-8.5%. Similarly, Poort et al. (2014) studied the effect of The Hague Court's decision of blocking The Pirate Bay, a pirate website based on the BitTorrent protocol on the percentage of other unlicensed platforms. They found that there is no blocking effect but, instead, downloading remained constant (a relapse effect).

Our paper is related to the recent theoretical and empirical literature on ad-blockers. Anderson and Gans (2011) provided one of the first analysis on ad-blockers, showing that, when consumers deviate from ads by purchasing an ad-blocker at a lower price, the content providers react by increasing the level of ads generating a potential circulation spiral and exploiting less (ads)sensitive customers. Their paper also showed that, by syphoning ads, incentives to invest in quality improvements are reduced, thereby altering the segment of consumers to which programs (or movies) are offered (from niched to general audience). In our paper, however, we depart from this strand of literature by considering a benevolent and 'free of charge ad-blocker', which nowadays simply takes

the form of a plug-in for browsers. Johnson (2013) modelled the strategic interaction between consumers and a firm who chooses the level of ads and the ads avoidance strategy. In contrast, in our model all consumers may adopt ad-blockers that are ‘free of charge’ and relatively simple to install. Another closely related empirical work is by Shiller et al. (2017), who using data from 2013 to 2016, studied the effect of ad-blocker adoption on website revenues. They found that ad-blocker adoption generates both reduction in revenues as a consequence of blocking ads and a significant reduction in the number of site visitors (with a marginal effect of 0.67%), undermining investments and websites’ quality in the long-run. Their conclusion that ad blocking poses substantial threats to ad-supported web is in harmony with our theoretical results.

3 The Model

We consider a fully covered market for piracy competition with vertical and horizontal product differentiation as in Chang and Walter (2015). Since our aim is to investigate the effect of ad-blocker adoption in the presence of single-product competition between a legal platform and a pirate hosting network, we rely on a stylised and simple model, abstracting from network effects and externalities¹².

3.1 Users’ Utility

Let v denote the standalone utility of visiting the (legal) platform which also represents the quality of the information good shown on this platform. We assume that the pirate version is an imperfect substitute of the original product and the degree of quality deterioration from the original product is captured by a scalar $a \in (0, 1)$ such that as $a \rightarrow 0$, the quality of the pirate version approaches the one sold by the legal firm, whereas as $a \rightarrow 1$, the quality of the pirate version is almost zero. Since, $a < 1$, the quality of the pirate product is always lower than the original one. This is quite intuitive for legal streaming platforms such as Netflix which offer several useful features like the

¹²See Belleflamme and Peitz (2010) for references on the importance of network effects. These are likely to influence the choice of a legal streaming service that, offering a broad range of movies or all the episodes or series, creates informational advantages over pirate streaming services, reducing searching costs and avoiding the risk of missing episodes, or seasons of the most preferred tv-series. In addition, network effects are likely to generate economics of scale as paying a flat price, the actual price per product decreases with the number of products, whereas this does not happen for pirate platforms, whose searching costs and advertisements are not discounted. However, considering these effects goes beyond the purposes of this study as we are concerned about per-product competition.

possibility of watching videos in different languages, the possibility to turn subtitles on or off, the possibility to restart from a certain point following an interruption, and high qualities of audios and videos. Such features are less likely to be found in pirate streaming platforms. Moreover, the possibility of finding movies dubbed or subtitled in languages other than English (American or British) or the original one is also lower in pirate platforms.

We assume that the potential users are uniformly distributed along a line of unit length as in a traditional Hotelling-style spatial model where x represents the ‘location’ of a potential user along the line. The opportunity cost of ‘travelling’ is t , the horizontal product differentiation parameter. We assume that the platforms are located at the extreme ends of the linear city where the legal platform (henceforth LP) is located at ‘0’ and the pirate platform (henceforth PP) is located at 1, and that the opportunity cost of *travelling* is linear in distance. To access the LP, users must pay a price p while the pirate service is offered free of charge. However, there are certain other frequently encountered non-monetary costs associated with using the pirate services such as

1. *Search costs*: there are search and learning costs for getting to the (appropriate) pirate platform (henceforth PP). We denote these costs by the parameter s ;¹³
2. The *annoyance cost*: this arises due to too many advertisements being shown on the PP.

The search cost arises as the link to the platform hosting the pirate content is provided by different (indeed thousands of) third party websites which create a (sort of) catalogue of all available pirate movies¹⁴. The second type of costs, the annoyance cost, arises as advertisements, while make the business sustainable in the absence of subscription fees, are often invasive and tedious for users, requiring them to close pop-ups and ads that often exhibit porn, dating, betting and online poker websites, or expose to malwares and fake softwares. To capture this increasing level of annoyance, we assume that the annoyance cost function $\Gamma(A)$ is a quadratic function of the number

¹³While we assume s to be exogenous in this paper, s can indeed be a choice variable. This remains to be explored in the future.

¹⁴Third-party websites work as intermediate between the user and the illegal content. Sometime a specific content is present in multiple illegal hosting networks and third party websites gather all these information, even by providing alternatives to link previously blocked by authorities. Thus, the user usually spends time (searching cost) to get to the pirate streaming hosting network.

of advertisements shown i.e. we assume:

$$\Gamma(A) = \gamma \frac{A^2}{2}, \quad \gamma > 0$$

where γ represents the annoyance cost parameter, assumed to be same for the entire population,¹⁵ and A denotes the number of advertisements shown.

When the market is fully covered, the utility of a user is as follows:

$$U_i = \begin{cases} v - p - tx, & \text{if going to the LP} \\ v(1 - a) - s - \gamma \frac{A^2}{2} - t(1 - x), & \text{if going to the PP} \end{cases} \quad (1)$$

Contrary to Chang and Walter (2015) who use an exogenous level of advertisements, in our model we assume that the illegal platform can decide on the amount of advertisements to be shown on its website. Thus in our model, the legal firm and the pirate firm compete with one another with respect to the price and the number of advertisements to be shown.

3.2 Demand

The market share of each platform is determined by solving the following marginal consumer's problem:

$$v - p - tx = (1 - a)v - s - \frac{\gamma A^2}{2} - t(1 - x)$$

Solving for x , the demand functions for each platform are given by:

$$\text{LP:} \quad D_L \equiv x = \frac{1}{2} + \frac{av - p + s}{2t} + \frac{\gamma A^2}{4t} \quad (2)$$

$$\text{PP:} \quad D_P \equiv 1 - x = \frac{1}{2} - \frac{av - p + s}{2t} - \frac{\gamma A^2}{4t} \quad (3)$$

3.3 The profit functions

The LP's profit function

¹⁵It will be interesting to consider heterogenous population with γ_i representing the annoyance cost parameter for the i -th individual. This will be useful in modelling consumers' decision to turn on or turn off the ad-blocker, and remains as a future research agenda.

The legal platform maximizes profits by choosing p . We assume that the marginal cost of producing the original information good is zero (a simplifying assumption) but there is a fixed cost F associated with its production. Thus, the LP's payoff function is:

$$\Pi_L = pD_L - F \quad (4)$$

It is easily verified that the above profit function is continuous and concave and hence twice differentiable.

The PP's profit function

The pirate platform maximizes profits through a sponsored-business model by deciding on the amount of advertisement A to be shown on consumers' screen. The model works just like a standard price competition case, as the willingness-to-pay for movies of consumers is monetised through advertisements. The revenue per advertisement is r , and is exogenous to the pirate firm since a single company has no bargaining power relative to the advertisement industry as a whole¹⁶. There are however fixed costs K for maintaining the system working (server, etc.)¹⁷: these fixed costs are strictly positive i.e. $K > 0$. As in the LP's case (and in line with the existing literature), marginal costs are set to zero. Thus the profit function of the PP is given by:

$$\Pi_P = D_P(rA) - K \quad (5)$$

3.4 Types of games

We consider a sequential-move game between the LP and the PP in the following types of markets:

The pre-adoption market where the LP and the PP engage in a sequential-move price-advertisement competition with no ad-blocker adoption. We call this the 'No ad-blocker adoption' game.

¹⁶In this sense, the platform is a price taker. For example, if the illegal platform decides to use non-aggressive advertising, as those provided by Google AdSense, it earns a fixed amount of money per thousands of visits.

¹⁷The presence of fixed costs is quite relevant because Megaupload and its "Mega Conspiracy" spent millions of dollars per month for supporting the business. Megaupload was also used to reward users uploading popular contents and contributing, indeed, to make the network more powerful and extended.

The post-adoption market where the ad-blocker decides on the "acceptable" level of advertisements to be shown on consumers' screen i.e the ad-blocker replaces the PP in this game making PP take a back-seat position. We call this the "Ad-blocker adoption game".

For each type of market, we alternate between the sequence of moves as below:

The LP game where the LP acts as a Stackelberg leader deciding on the price p , and the PP (or the ad-blocker) acts as a follower deciding on the level of advertisements A , after observing p ;

The PP game where the PP or the ad-blocker, acts as the Stackelberg leader choosing A , and the LP acts as a follower deciding on p , after observing the level of advertisements A .

We allow the ad-blocker in our model to pursue maximisation of two slightly different (though not unrelated) types of the objective function. In one scenario, we assume that the ad-blocker is fully benevolent and maximizes the consumer surplus under the condition that the pirate firm's profits be non-negative. This *constrained* optimisation problem allows us to study the consequences of partial blocking of advertisements better, which perhaps is the most realistic case as recently some ad-blockers have indeed made some changes in their strategies allowing for some of the contents to be shown on their websites. For example, Ad-block Plus, one for the most popular ad-blockers, created the "Acceptable Ads Initiative" to "allow advertisers and publishers who have agreed to make ads that abide by user-generated criteria to be white-listed"¹⁸.

In the other scenario, we assume that the ad-blocker pursues only an *unconstrained* maximisation problem whereby it merely maximises consumers' surplus without taking into account any effects on the PP's profit level. Note that the role played by the benevolent ad-blocker in this scenario is somewhat different from that of a typical welfare government often considered in the piracy literature.¹⁹ By maximising only the consumer surplus, the ad-blocker is not concerned (either partially or fully) about any

¹⁸The "Acceptable Ads initiative" defines unacceptable, among the others, pop-ups, pop-unders, rich media ads, pre-roll video ads and other invasive forms which are quite common in pirate hosting platforms. See <https://adblockplus.org/acceptable-ads>. Last visit on 16 October 2016.

¹⁹For example, Lopez-Cunat and Sanchez-Martinez (2015) used a semi-welfare function consisted in the consumer surplus and the profits of the legal firm. However, they showed that similar results apply, even when the welfare function also contains the profit surplus of pirate firms. In Banejee (2006), the government maximizes a welfare function containing profits of both firms (legal and pirate).

externalities created on other firms operating in the market, whereas a typical welfare-maximiser takes into consideration payoffs of all the actors in the market. Our approach however is consistent with the growing concern arising in the advertisement-based industry regarding the economic consequences of ad-blockers adoption (e.g. Forbes, a business magazine, installed ad-blocker walls).

The following assumption ensures that the results of our model are real-valued:

Assumption 1: $av + s < t$

This assumption, other than ensuring tractability of the model, implies that the mere presence of search costs and the quality degradation do not discourage consumers from watching movies on the pirate platform as the opportunity costs of attending a certain platform can be high.

4 The LP game

4.1 The No Ad-Blocker Adoption Game

Here, the LP sets p in the first stage of the game and the PP sets A in the second stage. We solve the model by backward induction starting with the PP's problem in the 2nd stage.

The second stage

Combining equation (3) and (5), the objective function of the PP is as follows:

$$\max_A \Pi_P = \left(\frac{1}{2} - \frac{av + s - p}{2t} - \frac{\gamma A^2}{4t} \right) rA - K \quad (6)$$

The first order condition (FOC) is then:

$$\frac{\partial \Pi_P}{\partial A} = 0 \Rightarrow -\frac{\gamma r A^2}{2t} + r \left(\frac{1}{2} - \frac{av + s - p}{2t} - \frac{\gamma A^2}{4t} \right) = 0 \quad (7)$$

Given p , the above FOC yields two solutions for A :

$$A_1 = \frac{\sqrt{6\gamma(t - (av + s) + p)}}{3\gamma} \quad A_2 = -\frac{\sqrt{6\gamma(t - (av + s) + p)}}{3\gamma} \quad (8)$$

Obviously we do not consider a negative value of A ; hence the only meaningful solution is represented by A_1 .²⁰ We call $A_1 = A$ unless otherwise stated. Concavity requirements are satisfied²¹.

Observation 1: *In the absence of ad-blockers, the optimal amount of advertisements decreases with the parameters of annoyance cost γ , the searching costs s , and the quality degradation a .*

Proof. Differentiating the optimal advertisement level with respect to the annoyance cost, it yields

$$\frac{\partial A}{\partial \gamma} = \frac{av + s - t - p}{\gamma \sqrt{6\gamma(t + p - (av + s))}} < 0 \quad (9)$$

whose numerator is negative and the denominator is positive by Assumption 1. Negative effects of quality degradation and search costs are evident from the numerator of A .

■

Lemma 1: *Advertisements and prices are strategic complements.*

Proof. Follows immediately from equation (8) as $\frac{\partial A}{\partial p} = \frac{\sqrt{6}}{6\sqrt{\gamma(t - (av + s) + p)}} > 0$. ■

The co-movement of prices and advertisements indicate that a reduction in the price charged by the LP forces the PP to strategically reduce its level of advertisements so as to limit or neutralize the *business stealing* effect coming from the legal provider. Similarly, an increase in the LP's price switches demands from the LP to the PP prompting the PP to raise its advertising level.

Lemma 1 is also crucial for studying the optimal response of the legal firm when ad-blockers are introduced: as ad-blockers (are supposed to) limit the amount of advertisements shown to users, it implies that prices for the LP should be lowered too although the extent of the reduction will depend upon the characteristics of the market and which platform leads the game.

²⁰Assumption 1 suffices to ensure that A_1 is real: As p is non-negative, the term under the square root is positive by assumption 1.

²¹The second order conditions (SOC) are negative $-\frac{3Ar\gamma}{2t} < 0$ because all its terms are positive.

The First Stage

Here, the LP acting as a leader, sets the optimal price anticipating the pirate firm's response. Rearranging the objective function in equation (4), the LP's problem is:

$$\begin{aligned} \max_p \Pi_L &= \left(\frac{1}{2} + \frac{av + s - p}{2t} + \frac{\gamma A^2}{4t} \right) p - F \quad s.t. \quad A = \frac{\sqrt{6\gamma(t - (av + s) + p)}}{3\gamma} \\ &= \left(\frac{2}{3} + \frac{(av + s) - p}{3t} \right) p - F \end{aligned} \quad (10)$$

The FOC is:

$$\frac{\partial \Pi_L}{\partial p} = 0 \Rightarrow \frac{2}{3} + \frac{av + s - 2p}{3t} = 0 \implies p^L = t + \frac{s + av}{2} \quad (11)$$

The optimal price charged by the LP, henceforth denoted by p^L , is the one charged in the Hotelling framework (opportunity cost to join the platform) plus a mark-up reflecting the magnitude of the quality degradation and the cost experienced for searching the movie. This finding indicates that the legal streaming platform strategically increases its price as a consequence of marginal increases in the searching costs. An implication of this result is that official anti-piracy measures aimed at blocking illegal website thereby (subsequently) increasing searching costs, relax the competition between legal and pirate firms, with the former increasing its price.²² In the worst scenario for the PP platform, *i.e.* when $a \rightarrow 1$ (*i.e.* the PP provides a "poor" substitute), other things being equal, the LP exerts its market power and charges the highest possible price: $\lim_{a \rightarrow 1} p^L = t + \frac{s+v}{2}$. Conversely, for $a \rightarrow 0$, when qualities are almost perfect substitutes, the competition becoming fiercer, induces the LP to reduce its price to $\lim_{a \rightarrow 0} p^L = t + \frac{s}{2}$

Substituting p^L from (11) into equation (8), we obtain the optimal level of adver-

²²Despite the fact that we consider exogenous searching cost, these can be influenced by anti-piracy policies operated by governments. Enforcing the removal of some contents (making the magnet link no longer available) without fully blocking the access to the pirate platform is common in the presence of copyright infringements. In these cases, the pirate video content is temporary removed and uploaded elsewhere, with a different magnet link. However, third party websites linking to the content might not promptly updated their pages to the new hosting link, increasing, indeed, the searching time for the users. In some other cases, when the legal hosting platform does not remove illicit materials, some broadband companies might inhibit the access to their customers, inducing pirate users to search for other streaming sources. *e.g.* Sky Broadband in the UK blocked the access to Rojadirecta (a platform streaming football matches), PopcornTime (providing access to tv-series) and many other websites under order of the High Court. See: <https://www.sky.com/help/articles/websites-blocked-under-order-of-the-high-court>. Last visit on 21 October 2016.

tisements, A^L :

$$A^L = \frac{\sqrt{3\gamma(4t - (av + s))}}{3\gamma} \quad (12)$$

Note, as in Observation 1, A^L is increasing with the opportunity cost t and decreasing with the parameters of search costs s , quality degradation a and the annoyance cost γ .

Finally, the resultant market shares, are:

$$D_L = \frac{1}{3} + \frac{av + s}{6t} \quad D_P = 1 - D_L = \frac{2}{3} - \frac{av + s}{6t} \quad (13)$$

Observe that, as in a traditional leader-follower game, moving first penalizes the LP in terms of demand *i.e.* $D_L < D_P$. To see that, subtract the pirate market share from that of the leader to yield: $D_L - D_P = \frac{1}{3} + \frac{av+s}{6t} - \left(\frac{2}{3} - \frac{av+s}{6t}\right) = -\frac{t-(av+s)}{3t} < 0$, as the numerator is positive by Assumption 1.

Finally, using the equilibrium values of p , the level of advertisements and the realised demand, the Nash values of profits are determined as follows:

$$\Pi_L = \frac{(av + s + 2t)^2}{12t} - F \quad \Pi_P = \frac{2}{3} - \frac{r(4t - (av + s))\sqrt{3\gamma(4t - (av + s))}}{18t\gamma} - K \quad (14)$$

In both cases profits are positive so long as the revenues compensate for the fixed costs.

4.2 The Ad-Blocker Adoption Game

Consider now the adoption of the ad-blocker. With the adoption of an ad-blocker, the role of the pirate firm is minimised, *i.e.* it is now the ad-blocker who is solely responsible for determining the number of advertisements to be filtered through the website: this makes the PP a completely passive actor. It is as if the ad-blocker has completely replaced the PP firm in the leader-follower game. Note that in the real-world, whilst ad-blockers can (fully or partially) block advertisements to be shown on websites, they first need to be downloaded and enabled by users. In our model, we have assumed that all consumers (who are taken to be homogenous with respect to their opportunity costs of time (t) and tolerance level) always switch on ad-blockers when this option is available as long as the post-adoption level of advertisements is lower than that in the case pre-adoption case. However, this need not be so. It will indeed be interesting to analyse heterogeneous consumers' decisions as to when to turn on ad-blockers - this

remains as an interesting avenue for our future research²³.

The ad-blocker, acting as a follower, makes its decision about A after observing the price set by the leader (the LP). As previously mentioned, we consider two slightly different scenarios for the ad-blocker's maximisation problem:

- (i) **The constrained maximisation problem** - the ad-blocker maximizes the aggregate consumer surplus with a non-negativity constraint on the pirate firm's profits; and
- (ii) **The unconstrained problem** - the ad-blocker simply maximizes the aggregate consumer surplus without any constraints.

We analyse these cases in turn.

4.2.1 The Constrained Problem

In this case, the ad-blocker solves the following problem:

$$\max_A CS = CS_P + CS_L \quad s.t. \quad \Pi_P \geq 0 \quad (15)$$

where

$$CS_L = \int_0^{D_L} [v - p - tx]dx, \text{ and } CS_P = \int_{D_L}^1 [(v(1 - a) - s - \frac{\gamma A^2}{2} - t(1 - x))]dx$$

$\Pi_P = \left(\frac{1}{2} - \frac{av+s-p}{2t} - \frac{\gamma A^2}{4t}\right) rA - K$ as given by equation (6), and D_L is given by equation (2). The Lagrangean for this problem is: $\mathcal{L} = CS + \lambda \Pi_P$ where $\lambda \geq 0$ is the Kuhn-Tucker multiplier. Then, the Kuhn-Tucker (K-T) necessary conditions for a maximum are:²⁴

²³If it is true that the adoption rate of ad-blockers is growing exponentially and firms are experiencing losses due to the cap on one of the sides of the market, it is also true that full coverage of the internet population is less likely to happen and that many websites pay for getting into the white-list ensuring fair advertisements or install anti ad-blocker systems (which increase the annoyance cost without hurting the level of ads in the web-page). We can also speculate about the fact that if a consumer has a certain level of familiarity with pirate networks, surfacing websites looking for the most preferred network, then she should have not too many problems in installing a simple plug-in often present among the extensions of the browser. In other words, those choosing the pirate network may have a certain background and knowledge of internet that would make them to adopt ad-blockers.

Also see our comment under footnote 13.

²⁴See the Appendix for derivations of the Consumer Surplus.

$$\begin{aligned}
& i) \quad \frac{\partial \mathcal{L}_A}{\partial A} = \frac{\partial CS}{\partial A} + \lambda \frac{\partial \Pi_P}{\partial A} \leq 0 \quad \text{for } A \geq 0 \\
\Rightarrow & \frac{\gamma A(A^2 \gamma + 2(av + s) - 2(p + t))}{8t} + \lambda r \left[\frac{1}{2} - \frac{av + s - p}{2t} - \frac{3\gamma A^2}{4t} \right] \leq 0 \quad \text{for } A \geq 0 \\
& ii) \quad \Pi_P = \left[\frac{1}{2} - \frac{av + s - p}{2t} - \frac{\gamma A^2}{4t} \right] rA - K \geq 0 \\
& iii) \quad \lambda \Pi_P = 0 \quad \lambda \geq 0
\end{aligned} \tag{16}$$

First, note that A must be positive. Otherwise with $A = 0$, Π_P becomes negative and hence the non-negativity constraint cannot be satisfied. Second note, that λ must be positive. We show this by contradiction. Suppose not. Suppose $\lambda = 0$. Then the above problem becomes an unconstrained problem which implies $\frac{\partial \mathcal{L}_A}{\partial A} = \frac{\partial CS}{\partial A} = 0$ as $A > 0$. This then yields $\hat{A} = \frac{(\sqrt{2\gamma(t-(av+s)+p)})}{\gamma}$ - see also the solution for the unconstrained problem in the next section. However, $\Pi_P|_{\hat{A}} < 0$ which violates the non-negativity constraint. Hence, λ must be strictly positive $\Rightarrow \Pi_P = 0$ in equilibrium. Intuitively, when the ad-blocker takes the pirate firm's profit into account, it allows just enough A to be passed through the pirate screen in order to keep the PP 'alive' while at the same time maximising overall consumer's surplus. Equilibrium level of advertising in this case, denoted by A^{L-BL} , is then obtained from solving the following:

$$\Pi_P = 0 \Rightarrow \left[\frac{1}{2} - \frac{av + s - p}{2t} - \frac{\gamma A^2}{4t} \right] A = \frac{K}{r}$$

However, given that it is difficult to find explicit solution from the above equation, we note that the above implies:

$$\left[\frac{1}{2} - \frac{av + s - p}{2t} - \frac{\gamma A^2}{4t} \right] > 0 \Rightarrow \frac{2(t - (av + s) + p)}{\gamma} > A^2$$

Let β , $0 < \beta < 1$, denote the scalar such that:

$$\frac{2(t - (av + s) + p)}{\gamma} \beta = A^2$$

Then,

$$A^{L-BL} = \frac{\sqrt{2\gamma\beta((p+t) - (av+s))}}{\gamma} \text{ such that } \Pi_P(A(\beta)) = 0 \quad (17)$$

Noting that $[\frac{1}{2} - \frac{av+s-p}{2t}]$ represents the extent of the PP's demand if there were no annoyance costs arising out of excessive A , β therefore represents the *proportion* of (or the extent of) 'no-annoyance' demand that now goes to A^{L-BL} . Thus higher β implies (as if) higher is the level of A as a proportion to the no-annoyance demand. As we shall see below, this parameter β representing the magnitude of no-annoyance demand now devoted to the level of A , plays a crucial role in determining how the LP prices its information good and how β now impacts consumers decision to turn the ad-blocker on or off and consequently the LP's profit.

The First Stage

Having determined the level of A chosen by the ad-blocker, we now turn to solving the leader's problem. In the first period, the leader (LP) solves the following problem:

$$\begin{aligned} \max_p \Pi_L &= \left[\frac{1}{2} + \frac{av+s-p}{2t} + \frac{\gamma A^2}{4t} \right] p - F \quad s.t. \quad A = \frac{\sqrt{2\gamma\beta(t - (av+s) + p)}}{\gamma} \\ &= \left[\frac{(1+\beta)t + (1-\beta)(av+s)}{2t} - \frac{(1-\beta)p}{2t} \right] p - F \end{aligned} \quad (18)$$

The relevant FOC is:

$$\begin{aligned} \frac{\partial \Pi_L}{\partial p} &= 0 : \frac{(1+\beta)t + (1-\beta)(av+s)}{2t} - \frac{(1-\beta)p}{t} = 0 \\ \implies p &= \frac{(1+\beta)t}{2(1-\beta)} + \frac{av+s}{2} \end{aligned} \quad (19)$$

Thus, we have

Lemma 2. *The greater the value of β , the greater is the price charged by the legal firm.*

Lemma 2 provides information about the relationship between the price p and β . Because the level of advertisements is increasing in β , the higher the β , the higher is the price since price and advertisements are strategic complements (Lemma 1).

Finally, substituting equation (19) into equation (17), the Nash equilibrium value of advertisement is:

$$A = \frac{\sqrt{\gamma \frac{\beta}{1-\beta} (t(3-\beta) - (av+s)(1-\beta))}}{\gamma}$$

Equilibrium market shares are then:

$$D_L^{L-BL} = \frac{(1+\beta)t + (1-\beta)(av+s)}{4t} \quad D_P^{L-BL} = 1 - D_L^{L-BL} \quad (20)$$

The post-adoption demand D_L^{L-BL} clearly increases with β and approaches half of the market for $\beta \rightarrow 1$, (although still suffering from the usual first-move disadvantage) as a higher value of β implies a higher proportion of ‘non-annoynce’ demand going to A which reduces PP ’s demand. The ultimate equilibrium value however is determined by the interaction of LP ’s pricing strategy along with other relevant parameters such as s, a and t . Finally, the legal platform’s profit equals

$$\begin{aligned} \Pi_L^{L-BL} &= \frac{(1+\beta)t + (1-\beta)(av+s)}{4t} \left[\frac{(1+\beta)t}{2(1-\beta)} + \frac{av+s}{2} \right] \\ &= \frac{((1+\beta)t + (1-\beta)(av+s))^2}{8t(1-\beta)} \end{aligned}$$

whilst the pirate profits equal zero.

Proposition 1. *When the legal platform acts as leader, the ad-blocker adoption reduces the demand for the legal firm compared to the pre-adoption case.*

Proof. Compare the LP demand pre- and post- ad-blocker adoption:

$$\begin{aligned} D_L - D_L^{L-BL} &= \frac{av+s}{6t} + \frac{1}{3} - \frac{(1+\beta)t + (1-\beta)(av+s)}{4t} \\ &= \frac{2(av+s) + 4t - 3(1+\beta)t - 3(1-\beta)(av+s)}{12t} \\ &= \frac{(1-\beta)(t - (av+s))}{12t} > 0 \end{aligned} \quad (21)$$

which is positive as $1 > \beta$ and $t > av + s$ (Assumption 1). ■

Proposition 1 indicates that the adoption of the ad-blocker has a *pro-piracy effect*, that increases the demand for the pirate platform at the expenses of the legal platform. The mere adoption of ad-blockers by consumers amplifies the substitutional effects, shifting part of the pre-adoption demand of the legal firm to the pirate competitor which becomes more attractive because of the reduced advertisements flow. The presence of substitutional effects arising with piracy is not new in the economic literature, with piracy usually displacing the demand for the original product when not working as sampling²⁵. The new advertisement level is lowered to the extent that it still preserves the existence of the pirate platform ($\Pi_P = 0$), but generates negative externalities (loss of demand) for the legal incumbent. The realised loss of demand decreases with β . The mechanism is as follows: as β increases two things happen: higher β means higher A which reduces the pirate firm's demand thereby increasing the legal firm's demand (direct effect); at the same time increase in A prompts the legal firm to increase its price p which depresses LP's demand and increases PP's demand (the strategic effect). The direct effect dominates, hence the overall LP's demand is higher compared to the pre-adoption case.

Obviously, the equilibrium value of A depends crucially upon the magnitude of the fixed cost K (as well as other other parameters such as r, v, a, s, t etc) such that $\Pi_P = 0$ holds.²⁶

Proposition 2. When $\beta > \frac{1}{3}$ consumers neglect to use ad-blockers whereas consumers turn on ad-blockers whenever $\beta \leq \frac{1}{3}$. Furthermore, when the legal platform acts as leader, the adoption of ad-blockers forces the LP to reduce its price which also hurts

²⁵See Peitz and Waelbroeck (2006a) on sampling in presence of free availability of digital contents.

²⁶E.g. to see how K affects A , rewrite $\Pi_P = 0$ as $\left[\frac{1}{2} - \frac{av+s-p}{2t} - \frac{\gamma A^2}{4t}\right] A = \frac{K}{r} \Leftrightarrow D_P(A)A = \frac{K}{r}$. Now, totally differentiate the above expression with respect to A and K to obtain:

$$\begin{aligned} A \frac{\partial D_P}{\partial A} dA + D_P(A) dA &= \frac{1}{r} dK \\ \Rightarrow dA \left[A \frac{\partial D_P}{\partial A} + D_P(A) \right] &= \frac{1}{r} dK \\ \Rightarrow \frac{dA}{dK} &= \frac{1}{r} \left[\frac{A}{D_P(A)} \frac{\partial D_P}{\partial A} + 1 \right] D_P(A) \end{aligned} \tag{22}$$

where $\frac{\partial D_P(A)/D_P(A)}{\partial A/A}$ represents the advertisements elasticity of demand. Since $D_P(A) > 0$ and $\partial D_P/\partial A < 0$, $dA/dK > 0$ if and only if the advertisement demand is inelastic i.e. $\left| \frac{\partial D_P(A)/D_P(A)}{\partial A/A} \right| < 1 \Rightarrow$ to maintain a non-negative profit, A can be increased when K increases only when the demand is inelastic.

its profits.

Proof: see the Appendix.

Proposition 2 which complements Proposition 1 says that consumers turn on ad-blockers whenever $\beta \leq \frac{1}{3}$, whilst they do not turn on ad-blockers when $\beta > \frac{1}{3}$. At first, Proposition 2 may seem counter-intuitive: given that A increases with β , why would consumers not turn on ad-blockers when the level of A is high enough? But this simply says how effective ad-blockers are in curbing nuisance advertisements: if the ad-blocker can successfully keep the level of advertisements down ($\beta \leq \frac{1}{3}$), then installing it makes sense for the consumers who then prefer to go onto the the pirate platform which then reduces the LP's demand and consequently its payoff. If however, the ad-blocker is not effective enough to keep the level of A low enough (i.e. $\beta > \frac{1}{3}$), then there is no point for the consumers to turn ad-blockers on. Therefore $\beta > \frac{1}{3}$ is the case which arises when the level of advertisements induced is larger than the pre-adoption case. The analysis of the effect of the ad-blocker adoption therefore holds for values of $\beta \in (0, \frac{1}{3}]$, whereas for any $\beta > \frac{1}{3}$, results from the pre-adoption scenario hold (Section 4.1).

4.2.2 The Unconstrained Problem

Now suppose that the ad-blocker simply maximises aggregate consumers' surplus i.e. the problem of the ad-blocker is now given by

$$\max_A CS$$

where $CS = CS_P + CS_A$ and as before

$$CS_L = \int_0^{D_L} [v - p - tx]dx, \text{ and } CS_P = \int_{D_L}^1 [(v(1 - a) - s - \frac{\gamma A^2}{2} - t(1 - x))]dx$$

The FOC then yields the following three solutions

$$A_1 = 0 \quad A_2 = \frac{\sqrt{2\gamma(t - (av + s) + p)}}{\gamma} \quad A_3 = -\frac{\sqrt{2\gamma(t - (av + s) + p)}}{\gamma} \quad (23)$$

of which only the first two are meaningful. The solution $A_1 = 0$ refers to the situation where the ad-blocker obstructs *all* advertisements, making consumers not experience *any* annoyance cost. The second solution A_2 refers, instead, to the case with enhanced

advertising as discussed below²⁷. Assumption 1 is crucial in ensuring that A_2 is not a complex solution but real. This leads us to the following proposition.

Proposition 3. *When ad-blockers are activated and maximise only the consumer surplus, both the equilibrium solutions of A lead to foreclosure.*

Proof. Consider Equation (23), $A_1 = 0 \implies \Pi_P < 0$, which clearly implies that the PP does not operate. Next, consider A_2 and substitute it into equation (2), then $D_L = 1 \implies D_P = 0$ and all market shares go to the LP. In both cases, the pirate firm is driven out of the market. ■

Proposition 3 illustrates some strong and interesting results, highlighting the implications of the unconstrained optimisation problem: when ad-blockers care *only* about consumers' welfare, their strategies can drive pirate firms out of the market thereby establishing a monopoly position of the legal firm! We consider the above results in more details below.

Full Blocking: $A_1 = 0$

In this case, pirate firms can never cover their fixed costs and hence exit the market as their profits otherwise go negative²⁸. Thus, the LP attains the monopoly position in the market. Interestingly, as the following corollary shows, even when the LP becomes monopoly, the market is not fully covered.

Corollary 1. *In the unconstrained problem, when ad-blockers fully shut-down advertisements and the LP acts as a leader, the market is never covered and the price charged to LP users shrinks to $p = \frac{t+(av+s)}{2}$.*

Proof: see the Appendix.

Full blocking thus represents a drastic solution that works as an extreme anti-piracy measure (the PP completely shutting down due to negative profits). Corollary 1 explores some counter-intuitive implications of full blocking: As the platform faces no rivalry, we might expect all consumers moving to the LP. However, as part of the demand D_L is actually generated by advertisement substitution effects, absent advertisements, D_L is reduced and some consumers do not visit the platform although the

²⁷Note that $\lim_{t \rightarrow av+s} A_2 = A_1 = 0$, as a consequence of the fact that when the opportunity cost is at its lower bound ($t = av + s$), $A_2 = 0$ and there is no partial blocking.

²⁸The same result prevails when there are no fixed costs but non-negative marginal costs of production.

price charged to LP customers is lower than that under competition. Specifically, when the LP in the first period maximizes profits, it charges a price of $p = \frac{t+(av+s)}{2}$ entailing the following market share:

$$D_L = \frac{1}{4} + \frac{av + s}{4t} < 1$$

Comparing the resulting price with the pre-adoption one (Equation (11)), the latter is higher than the former, indicating that, despite the price reduction, some PP consumers do not join the LP when piracy is completely fought. This is also clear from the change in the demand for the LP: comparing $D_L^{L-Noads} = \frac{1}{4} + \frac{av+s}{4t}$ and that under pre-adoption (Equation (13)),

$$D_L^{L-No-ads} - D_L = -\frac{t - (av + s)}{12t} < 0$$

This result is mainly dependent on the strategic complementarity of prices and advertisements: as $A \rightarrow 0$ so does p . Overall, reduced prices and lower demand hurt LP's profits compared to the pre-adoption case. Hence it is never profitable for the legal firm to have the PP completely drawn out of the market, as competition between the two generate induced-demand.

Partial Blocking: $A_2 > 0$

Now consider the case with partial blocking where the advertisement level is given by A_2 (see equation (23)). Here, $D_L = 1 \Rightarrow$ all consumers actually go to the LP. In this case, the level of advertisements is so high that it fully chokes off the demand for the pirate platform: the adoption of ad-blockers increases the annoyance cost of advertisements to such an extent that all consumers are induced to switch to the legal firm. Consequently, even with positive A_2 , the pirate firm ends up making negative profits (due to zero pirate demand and positive fixed costs) and hence exits the market.

Thus, partial blocking of A also entails a monopoly outcome for the legal firm. In this case, the incumbent exploits its market power by choosing any price bounded from below by the fixed cost F and bounded from above by consumers' net willingness to pay. Thus, the optimal price lies in the interval $F < p \leq v - t$, implying that a rational firm will choose the maximum possible price $p = v - t$. Interestingly, although the ad-blocker finds it optimal to increase the level of advertisements though not directly shutting down the pirate firm (Corollary 1), all consumers choose to join the legal incumbent which therefore exploits its market power by charging the highest possible

price²⁹. Hence we have,

Corollary 2. *In the unconstrained problem, when the legal platform moves as the leader, the ad-blocker increases the advertisement flows to such an extent that the market is fully served by the legal firm who then sets the price at its maximum $p = v - t$. Consequently, pirate consumers never use ad-blockers.*

Proof: see the Appendix.

The above analysis highlight some interesting and contrasting implications of ad-blockers adoption on the resultant market outcome, depending upon the objective function pursued by the ad-blocker. On one hand, in the constrained consumer surplus maximisation, the adoption of ad-blockers fosters piracy, shifting part of the LP demand to the PP platform, whose profits go to zero. The existence of piracy is, in any case, beneficial for consumers, which experience lower price and a low-cost alternative, according to their individual preferences. In addition to lower prices, consumers are protected from less competitive markets arising from monopoly outcomes. On the other hand, when ad-blockers care only about consumers (the unconstrained problem), regardless of its effects on profits of operating firms, either some consumers prefer to remain out of the market and not join the monopoly platform (Corollary 1) or they perceive the existence of ad-blockers as utility-diminishing (Corollary 2) and hence all the market demand is captured by the LP. Obviously, when consumers ignore the existence of ad-blockers, results from the pre-adoption case prevail (Section 4.1).

5 The PP-game

Now we consider the scenario where the pirate platform PP acts as the leader and the legal platform, after observing the behaviour of the PP, moves as the follower. Observing the behaviour of pirate rival is not new to digital companies providing streaming contents as is also reported by the Netflix Vice President³⁰. Previous papers on piracy and on leader-follower game showed that the incumbent prefers to act as the leader under all different anti-piracy systems (Banerjee 2003), whereas Martinez-Sanchez (2011)

²⁹This effect is also clear from Equation (A-2) in Appendix, with the consumer surplus of pirate consumers being increasing with A .

³⁰See <http://www.bbc.co.uk/news/technology-24108673>. Last visit on 15 October 2016.

showed that in a price competition framework the government makes the legal incumbent decide on whether to act as a leader or as a follower depending upon the magnitude of the piracy monitoring cost. In our model, we do not consider the intervention of a government. Instead, we rely only on the presence of a benevolent ad-blockers aiming to reduce the annoyance cost from advertisements. As in the LP game, here also we consider pre- and post-adoption outcomes and compare them.

The second stage is common for both scenarios: The LP acts a follower, choosing the price after observing the advertisement level in the market. I.e the LP's problem is:

$$\begin{aligned}\max_p \Pi_L &= D_L p - F \\ &= \left[\frac{1}{2} + \frac{av + s - p}{2t} + \frac{\gamma A^2}{4t} \right] p - F\end{aligned}$$

which yields the following FOC:

$$\frac{\partial \Pi_L}{\partial p} = 0 \Rightarrow \frac{1}{2} + \frac{av + s - p}{2t} + \frac{\gamma A^2}{4t} - \frac{p}{2t} = 0 \Rightarrow p(A) = \frac{s + av + t}{2} + \frac{A^2 \gamma}{4} \quad (24)$$

5.1 The No Ad-blocker Adoption Game

Consider first the pre-adoption case. In the first period, anticipating the optimal price, the PP decides the optimal level of advertisements:

$$\begin{aligned}\max_A \Pi_P &= D_P r A - K \quad s.t. \quad p = \frac{s + av + t}{2} + \frac{A^2 \gamma}{4} \\ &= \frac{6t - 2(av + s) - \gamma A^2}{8t} A r - K\end{aligned} \quad (25)$$

The first order condition is:

$$\frac{\partial \Pi_P}{\partial A} = 0 \Rightarrow -\frac{\gamma A^2 r}{4t} + r \frac{6t - 2(av + s) - \gamma A^2}{8t} = \frac{r(6t - 2(av + s) - 3\gamma A^2)}{8t} = 0 \quad (26)$$

which yields:

$$A_1 = \frac{\sqrt{6\gamma(3t - (av + s))}}{3\gamma} \quad A_2 = -\frac{\sqrt{6\gamma(3t - (av + s))}}{3\gamma} \quad (27)$$

As before, we restrict our analysis only on the positive root A_1 , which we call A^F .

Substituting for A^F into Equation (24), the equilibrium price is $p^F = t + \frac{(av+s)}{3}$, while the demand shares for the LP and the PP are, respectively, the following:

$$D_L^F = \frac{av + s}{6t} + \frac{1}{2} \quad D_P^F = \frac{av + s}{6t} - \frac{1}{2} \quad (28)$$

Using p^F and D_L^F , the profits are

$$\begin{aligned} \text{LP: } \Pi_L^F &= \left[\frac{av + s}{6t} + \frac{1}{2} \right] \left[t + \frac{(av + s)}{3} \right] - F \\ &= \frac{(3t + av + s)^2}{18t} - F \end{aligned}$$

$$\text{PP: } \Pi_P^F = \left[\frac{1}{2} - \frac{av + s}{6t} \right] \frac{r\sqrt{6\gamma(3t - (av + s))}}{3\gamma} - K$$

5.2 The Ad-blocker Adoption Game

We now introduce the ad-blocker, who moves first in the market, replacing the PP and taking the decision on A on behalf of the PP, who then simply acts based on the level of A chosen by the ad-blocker. As in the LP game, we consider two types of objective functions of the ad-blocker: (i) the constrained case where the PP maximizes the consumer surplus, subject to the condition that pirate profits are non-negative and anticipating the second period price; and the (ii) unconstrained case problem reported in the Appendix mirror our previous results.

5.2.1 The constrained problem

The problem faced by the ad-blocker in the first period is the following:

$$\begin{aligned} \max_A \quad CS \quad & s.t. \quad p = \frac{s + av + t}{2} + \frac{A^2\gamma}{4} \\ & \Pi_P \geq 0 \end{aligned} \quad (29)$$

which is solved by deriving the Kuhn-Tucker conditions for a maximum (see the Appendix). As in the previous case, let α denote the parameter, $0 < \alpha < 1$, such that $\Pi_P(A(\alpha)) = 0$ (see the appendix). The optimal level of advertisements under ad-blocker adoption then is:

$$A^{F-BL} = \frac{\sqrt{2\gamma\alpha(3t - (av + s))}}{\gamma} \quad (30)$$

Note, α has the similar interpretation as β under the LP game i.e. α represents the *proportion* of (or the extent of) ‘no-annoyance’ demand in this case when the ad-blocker (or the PP) acts as the leader that now goes to A^{F-BL} . Thus higher α implies a higher level of A as a proportion to the no-annoyance demand. Substituting the resultant ‘scaled’ A^{F-BL} into p^{F-BL} , the price charged by the LP is:

$$p^{F-BL} = \frac{s + av + t}{2} + \frac{\gamma A^2}{4} = \frac{t(1 + 3\alpha) + (av + s)(1 - \alpha)}{2} \quad (31)$$

The market shares are:

$$D_L^{F-BL} = \frac{t(1 + 3\alpha) + (av + s)(1 - \alpha)}{4t} \quad (32)$$

$$D_P^{F-BL} = 1 - \frac{t(1 + 3\alpha) + (av + s)(1 - \alpha)}{4t} \quad (33)$$

The overall profits are:

$$\begin{aligned} \text{LP} & : \quad \Pi_L^{F-BL} = \frac{(t(1 + 3\alpha) + (av + s)(1 - \alpha))^2}{8t} \\ \text{PP} & : \quad \Pi_P = 0 \end{aligned}$$

Proposition 4. *When the legal platform acts as follower, and the ad-blocker acts as a*

leader on behalf of the PP and pursues a constrained optimization problem, the pirate profit is reduced to zero whilst the demand, price and profit of the legal firm are lowered by the adoption of the ad-blocker compared to the pre-adoption levels.

Proof: see the Appendix.

Proposition 4 parallels our previous results described by Proposition 1 and 2. The mechanism behind these results is also similar as before. Despite the fact that the LP acting as a follower exploits its second-mover advantage, is made worse off compared to the pre-adoption scenario. The pro-piracy effect that increases the attractiveness of the pirate platform (because of its reduced advertisements) offsets the strategic effect of price reduction. Thus the LP suffers from the ad-blocker adoption.

5.2.2 The unconstrained problem

In this case, the ad-blocker's problem is similar to the previous case, *i.e.*

$$\max_A CS$$

From the first order conditions (see the appendix for derivation), we obtain

$$A_1 = 0 \quad A_2 = \frac{\sqrt{2\gamma(7t - (av + s))}}{\gamma} \quad A_3 = -\frac{\sqrt{2\gamma(7t - (av + s))}}{\gamma} \quad (34)$$

ignoring the negative value given by A_3 , the only meaningful solutions are

- (i) *Full blocking:* $A_1 = 0$. Here the outcome is the same as in the previous section (see Corollary 1 and the following discussion), where the equilibrium values of variables of the legal firm are:

$$p^F(A_1 = 0) = \frac{s + av + t}{2} \quad \text{and} \quad D_L^{F-Noads} = \frac{1}{4} + \frac{av + s}{4t} < 1$$

Consequently, the pirate platform is forced to exit the market due to the loss of revenues (in the presence of fixed cost K).

- (ii) *Partial blocking:* $A_2 > 0$. Calling A_2 as A^{F-BL} , we obtain that $A^{F-BL} = \frac{\sqrt{2\gamma(7t - (av + s))}}{\gamma} \geq A^F = \frac{\sqrt{6\gamma(3t - (av + s))}}{3\gamma}$.³¹ Therefore, an enhanced level of advertise-

³¹Again, this comparison between the pre- and post- adoption of ad-blockers advertising levels can be obtained using (as before) a non-linear monotonic transformation of power of 2, which lead the above inequality being true for any $9t > (av + s)$, which is always the case under Assumption 1.

ments relative to the pre-adoption case makes consumers not preferring using the ad-blocker at all - a similar implication as in Corollary 2 (when the legal platform was the leader). However, this immediately makes consumers not use the PP at all making $D_L^{F-BL} = 1 \Rightarrow$ the PP is forced out of the market. Hence we have the following corollary (stated without proof) summarising the above result.

Corollary 3. *In the unconstrained problem, when the ad-blocker moves as a leader and the legal platform as a follower, the ad-blocker chooses A either so low ($A_1 = 0$) or so high ($A > A|_{\text{pre-adoption}}$) that consumers neither use ad-blockers nor use the pirate platform which then entails the legal firm a monopoly position in the market triggering higher prices.*

Corollary 3 simply re-iterates the results described by Proposition 3 and Corollary 1-2. To summarise, we show that with the legal platform moving as a follower, when the ad-blocker pursues an *unconstrained consumer surplus maximisation* problem, two solutions can arise (full blocking and partial blocking of A), where in both of these cases consumers prefer **not** to turn ad-blockers on.

6 Conclusions

In this paper we have analysed the price-advertisement competition between a legal and a pirate firm in the video-on-demand market to examine the impact of ad-blocker adoption on digital piracy. Our results build upon some important and growing concerns about the recent trend of ad-blocker adoption: by directly damaging the advertisement industry and lowering the viability of sponsored-business models, the adoption of ad-blockers generates indirect effects on other industries thereby altering the functioning of the entire digital market.

We have shown that, depending upon the objective functions of the ad-blockers, very contrasting outcomes can prevail with severe consequences on the market structure. We have shown that when ad-blockers maximize consumers' surplus subject to the non-negativity constraint on the pirate platform's profit, not only does it lower the pirate platform's profit down to zero (though not necessarily inducing complete exit of the firm), it also generates negative spillovers on the legal competitors by displacing their demand thereby reducing their market profitability. Both firms' profits are hurt in this sense. On the other hand, when the ad-blockers are concerned *only* about maximising

consumers' surplus, they attempt to preserve consumers' welfare by either choosing a zero level of advertisement or a (an inefficiently) high level of advertisement such that consumers never use ad-blockers on the pirate platform - this reduces the pirate firm's profit down to zero and consequently induce them to exit! Ironically, this turns the duopoly market into a monopoly one where only the legal platform operates which then charges as high a price as admissibly possible (corollaries 1 and 2). Such outcomes not just arise from the nature of the price-ads competition but also to a large extent due to the externalities arising from the adoption of ad-blockers itself. In this sense, as the adoption of ad-blockers represents a threat for advertising revenues, a result which resonates well with Shiller-Waldfoegel-Ryan's (2017) recent empirical finding that "ad blocking poses a substantial threat to the ad-supported web".

On the video-on-demand sector like the one we have analysed in this paper, the presence of an illegal alternative poses a competitive threat to the legal firm, preventing price hike to the monopolist level. So implementing policies leading to subsequent exit of the pirate firm making piracy non-viable may not necessarily be the best policy. From a managerial point of view however, given that we find that in a price-advertisement competition the adoption of ad-blockers is likely to magnify the substitutional (pro-piracy) effects by displacing demands for original products, particular attention should be posed on the quality aspect. Specifically, legal platforms should try to differentiate their services relative to cyber-lockers and hosting networks as much as possible by providing better quality and features that are difficult to replicate e.g. user-friendly interfaces, multiple subtitles, wish-lists and such. This strategic differentiation is crucial for those video-on-demand providers for fostering growth other than just using pricing strategies to deter piracy. Alternatively, such strategic differentiation could also take the form of measures taken to increase consumers' search costs for pirate firms. Overall, 'mild' anti-piracy policies that do not fully shut down piracy, are perhaps preferable.

Our results also have important policy implications for *designing* suitable anti-piracy measures. We have shown that, on one hand, absent ad-blockers, consumers experience higher prices and increased annoyance costs, although the demand and revenues of the pirate firm are positive; on the other hand, with ad-blockers adoption, whilst consumers are better-off, the legal firm maybe worse-off along with the pirate firm whose profit goes to zero. Thus, depending upon the *perspective* and the *goal* of anti-piracy measures (i.e. cutting pirate demand or reducing its revenues), ad-blockers can either be regarded as a "blessing" or a "curse". If the anti-piracy measure is targeted towards cutting pirate demand, the government should focus on e.g. increasing search costs for users by seizing

websites and hosting network whilst still keeping piracy 'alive' instead of allowing ad-blockers to induce complete shut-down of pirate firms which endows monopoly crown to the legal firm. The latter perspective of ad-blockers adoption can be regarded as a curse! Conversely, if the objective is to block the flow of money into illegal (and, presumably, non-taxed) firms, ad-blocker adoption can be seen as a "blessing" as it can lead to zero profits earned by pirate firms. Therefore, both the government and the legal firm should carefully address the presence of this plug-in tool in designing their anti-piracy measures and their managerial strategies as the effect might not be unidirectional.

Appendix

Derivation of CS and its derivate in the sub-section 4.2.1

The total consumers' surplus $CS = CS_L + CS_P$. The surplus of consumers joining the legal firm:

$$\begin{aligned}
 CS_L &= \int_0^{D_L} [v - p - tx]dx = -\frac{x^2t}{2} + x(v - p) \\
 &= -\frac{x^2t}{2} + x(v - p) \Big|_0^{\frac{1}{2} + \frac{av-p+s}{2t} + \frac{\gamma A^2}{4t}} \\
 &= -\frac{(2(av - p + s + t) + \gamma A^2)^2}{32t} + \frac{2(av - p + s + t) + \gamma A^2}{4t}(v - p)
 \end{aligned}$$

Similarly, the surplus of those consumers joining the pirate platform:

$$\begin{aligned}
 CS_P &= \int_{D_L}^1 [v(1 - a) - s - \frac{\gamma A^2}{2} - t(1 - x)]dx \\
 &= \frac{x^2t}{2} + x((1 - a)v - s - \frac{\gamma A^2}{2} - t) \Big|_{\frac{1}{2} + \frac{av-p+s}{2t} + \frac{\gamma A^2}{4t}}^1
 \end{aligned}$$

which implies that $CS_P = F(1) - F(D_L)$, where

$$\begin{aligned}
 F(1) &= \frac{t}{2} + ((1 - a)v - s - \frac{\gamma A^2}{2} - t) \\
 F(D_L) &= \frac{(\frac{1}{2} + \frac{av-p+s}{2t} + \frac{\gamma A^2}{4t})^2 t}{2} + (\frac{1}{2} + \frac{av-p+s}{2t} + \frac{\gamma A^2}{4t})((1 - a)v - s - \frac{\gamma A^2}{2} - t)
 \end{aligned}$$

The partial derivatives

(i) The partial derivative of CS_L with respect to A is:

$$\begin{aligned}
 \frac{\partial CS_L}{\partial A} &= -\frac{(2(av-p+s+t)+\gamma A^2)\gamma A}{8t} + \frac{\gamma A}{2t}(v - p) \\
 &= \frac{A\gamma(4v-\gamma A^2-2(t+p)-2(av+s))}{8t}
 \end{aligned} \tag{A-1}$$

(ii) The related partial derivative of CS_P with respect to A is

$$\frac{\partial CS_P}{\partial A} = \frac{\partial F(1)}{\partial A} - \frac{\partial F(D_L)}{\partial A}$$

and subsequently by:

$$\begin{aligned}
\frac{\partial CS_P}{\partial A} &= -\gamma A - \left[\frac{\gamma A}{2t} \left(\frac{1}{2} + \frac{av - p + s}{2t} + \frac{\gamma A^2}{4t} \right) - \gamma A \left(\frac{1}{2} + \frac{av - p + s}{2t} + \frac{\gamma A^2}{4t} \right) \right] - \\
&\quad \frac{\gamma A}{2t} \left((1-a)v - s - \frac{\gamma A^2}{2} - 2 \right) \\
&= \frac{A\gamma(6(av + s) + 3A^2\gamma - 2(p + t) - 4v)}{8t}
\end{aligned} \tag{A-2}$$

Summing equation (A-1) and (A-2), then the derivative is:

$$\begin{aligned}
\frac{\partial CS}{\partial A} &= \frac{A\gamma(4v - \gamma A^2 - 2(t + p) - 2(av + s))}{8t} + \frac{A\gamma(6(av + s) + 3A^2\gamma - 2(p + t) - 4v)}{8t} \\
&= \frac{\gamma A(A^2\gamma + 2(av + s) - 2(p + t))}{8t}
\end{aligned} \tag{A-3}$$

Proof of Proposition 2

To effectively work, ad-blockers are supposed to reduce the level of advertisements as compared to the pre-adoption case. When this is not the case, PP consumers neglect to turn on the ad-blocker as it is utility diminishing. We prove Proposition 2 by contradiction. Suppose that the ad-blocker does not successfully work and it increases the amount of advertisements from A^L to A^{L-BL} , such that $A^{L-BL} \geq A^L$. Because both elements involve square roots, let us apply a quadratic monotonic transformation $f(A^{L-BL}) \geq f(A^L)$, such that $A^{L-BL} \geq A^L$ as long as the argument under the root is greater than 1³²:

$$\begin{aligned}
A^L \leq A^{L-BL} &\implies \frac{\sqrt{3\gamma(4t - (av + s))}}{3\gamma} \leq \frac{\sqrt{\gamma \frac{\beta}{1-\beta} (t(3-\beta) - (av + s)(1-\beta))}}{\gamma} \\
f(A^L) \leq f(A^{L-BL}) &\implies 4t - (av + s) \leq 3 \frac{\beta}{1-\beta} (t(3-\beta) - (av + s)(1-\beta)) \tag{A-4} \\
t(4 - 13\beta + 3\beta^2) &\leq (av + s)(1-\beta)(1-3\beta)
\end{aligned}$$

³²This is always the case as Assumption 1 ensures that $t > \frac{1}{12\gamma} + \frac{av+s}{4}$ (for A^L) and $t > \frac{1-\beta}{\gamma\beta(3-\beta)} + \frac{(av+s)(1-\beta)}{3-\beta}$ (for A^{L-BL}).

To make A^L real the underlying assumption is that $t > \frac{av+s}{4}$, entailing that $\frac{av+s}{4}(4 - 13\beta + 3\beta^2) < t(4 - 13\beta + 3\beta^2) \leq (av + s)(1 - \beta)(1 - 3\beta)$. This also implies that $\frac{(av+s)(4-13\beta+3\beta^2)}{4} < (av + s)(1 - \beta)(1 - 3\beta)$, which dividing by $av + s$ and multiplying by 4 it yields $(4 - 13\beta + 3\beta^2) < 4(1 - \beta)(1 - 3\beta)$. Solving for β , we obtain $3\beta < 9\beta^2 \implies \beta > \frac{1}{3}$. By contradiction, consumers find optimal to turn-on advertisements for any $\beta < \frac{1}{3}$ ³³. Instead, ad-blockers are ineffective and show the same (for $\beta = \frac{1}{3}$) or a greater (for $\beta > \frac{1}{3}$) amounts of advertisements than in the pre-adoption scenario. Next, to verify the effect on prices given that consumers either use or neglect ad-blockers. Comparing pre- and post- adoption prices:

$$p_L - p_L^{L-BL} = \frac{av + s}{2} + t - \left(\frac{t(1 + \beta)}{2(1 - \beta)} + \frac{av + s}{2} \right) = \frac{t(1 - 3\beta)}{2(1 - \beta)}$$

As $\beta < 1$ by definition, the difference in price is either negative or positive depending on the magnitude of β . If the scale parameter is sufficiently low such that $\beta < \frac{1}{3}$: $p_L > p_L^{L-BL}$. For $\beta = \frac{1}{3}$, prices are equal, whereas for any level of $\beta > \frac{1}{3}$, prices as a consequence of ad-blocker adoption are higher than in its absence. As for $\beta < \frac{1}{3}$ both demand and prices decrease, profits decreases too. This is analytically proven by comparing pre- and post-adoption profits. Call $\Pi_L^L - \Pi_L^{L-BL}$ the difference in profits, then it is equal to:

$$\begin{aligned} \Pi_L^L - \Pi_L^{L-BL} &= \frac{(av + s + 2t)^2}{12t} - \frac{((1 + \beta)t + (1 - \beta)(av + s))^2}{8t(1 - \beta)} \\ &= \frac{1}{36t(1 - \beta)} [t^2(13 - 2\beta - 3\beta^2) + (av + s)^2(1 + 2\beta - 3\beta^2) + 2t(av + s)(1 - 4\beta + 3\beta^2)] \end{aligned} \tag{A-5}$$

Suppose $\Pi_L^L - \Pi_L^{L-BL} < 0$ and $t^2(13 - 2\beta - 3\beta^2) + 2t(av + s)(1 - 4\beta + 3\beta^2) < -(av + s)^2(1 + 2\beta - 3\beta^2)$. As by Assumption 1 $t > av + s$, it must be true that $(av + s)^2(13 - 2\beta - 3\beta^2) + 2(av + s)^2(1 - 4\beta + 3\beta^2) < -(av + s)^2(1 + 2\beta - 3\beta^2)$. Dividing by $(av + s)^2$, we obtain $(13 - 2\beta - 3\beta^2) + 2(1 - 4\beta + 3\beta^2) < -(1 + 2\beta - 3\beta^2) \implies 2(8 - 4\beta + 3\beta^2) < 0$, which is never the case. By contradiction, it is therefore proven that $\Pi_L^L - \Pi_L^{L-BL} > 0$. ■

³³Results are robust also to the use of Assumption 1, which is more restrictive in this scenario.

Proof of Corollary 1

Consider the LP maximisation problem absent advertisements, taking the following form:

$$\begin{aligned} \max_p \Pi_L &= \left[\frac{1}{2} + \frac{av + s - p}{2t} + \frac{\gamma A^2}{4t} \right] p - F \quad s.t. \quad A_1 = 0 \\ &= \left[\frac{1}{2} + \frac{av + s - p}{2t} \right] p - F \end{aligned} \quad (\text{A-6})$$

The related FOC and the resultant optimal price are:

$$\frac{\partial \Pi_L}{\partial p} = \frac{1}{2} + \frac{av + s - p}{2t} - \frac{p}{2t} = 0 \quad (\text{A-7})$$

Rearranging, the optimal price is $p = \frac{t + (av + s)}{2}$. ■

Proof of Corollary 2

Compare A^{L-BL} and A^L (Equation (12)) with $p^{F-BL} = v - t$, so as to obtain $A^{L-BL} = \frac{\sqrt{2\gamma(v(1-a)-s)}}{\gamma}$, with $v(1-a) - s > 0$ by default and ensuring that the solution is not complex. Suppose that ad-blocker does not successfully work and increase the level of advertisements from A^L to A^{L-BL} , such that $A^{L-BL} \geq A^L$, then consumers neglect the existence of ad-blockers as utility-diminishing. As both elements involve square roots, let us apply a quadratic monotonic transformation $f(A^{L-BL}) \geq f(A^L)$ implies $A^{L-BL} \geq A^L$ if and only if the argument under the root is greater than 1. This condition is always satisfied for A^L as Assumption 1 ensures that $t > \frac{1}{12\gamma} + \frac{av+s}{4}$ and for A^{L-BL} as long as $v > \frac{1}{2\gamma} + (av + s)$. Therefore, comparing advertisements under both scenarios, then:

$$\begin{aligned} A^L \leq A^{L-BL} &\implies \frac{\sqrt{3\gamma(4t - (av + s))}}{3\gamma} \leq \frac{\sqrt{2\gamma(v(1-a) - s)}}{\gamma} \\ f(A^L) \leq f(A^{L-BL}) &\implies 4t - (av + s) \leq 6(v(1-a) - s) \\ &= 4t \leq 6v - 5(av + s) \end{aligned} \quad (\text{A-8})$$

As $t > \frac{av+s}{4}$ for ensuring real roots, then it also implies that $6v - 5(av + s) \geq 4\frac{av+s}{4}$. Simplifying, we obtain $6v - 5(av + s) \geq (av + s) \implies 6v \geq 6(av + s)$, which is always true. Hence, $A^L < A^{L-BL}$ such that ad-blocker increases the level of advertisements, and consumers prefer the pre-adoption level of advertisements A^L to the post-adoption

level A^{L-BL} . ■

Derivation of CS and its derivatives in the sub-section 5.2.1

$CS = CS_L + CS_P$ where

$$CS_L = -\frac{x^2t}{2} + x(v-p)\Big|_0^{\frac{1}{2} + \frac{av-p+s}{2t} + \frac{\gamma A^2}{4t}}$$

Using $p = \frac{s+av+t}{2} + \frac{A^2\gamma}{4}$ (Equation (27)), then

$$\begin{aligned} CS_L &= -\frac{x^2t}{2} + x\left(v - \frac{s+av+t}{2} - \frac{A^2\gamma}{4}\right)\Big|_0^{\frac{2(av+s+t)+\gamma A^2}{8t}} \\ &= -\frac{(2(av+s+t) + \gamma A^2)^2}{128t} + \frac{2(av+s+t) + \gamma A^2}{8t} \left(v - \frac{s+av+t}{2} - \frac{A^2\gamma}{4}\right) \end{aligned}$$

Next,

$$\begin{aligned} CS_P &= \frac{x^2t}{2} + x\left((1-a)v - s - \frac{\gamma A^2}{2} - t\right)\Big|_{\frac{2(av+s+t)+\gamma A^2}{8t}}^1 \Rightarrow F(1) - F(D_L) \quad \text{where} \\ F(1) &= v(1-a) - s - \frac{t}{2} - \frac{\gamma A^2}{2} \quad \text{and} \\ F(D_L) &= \frac{(2(av+s+t) + \gamma A^2)^2}{128t} + \frac{2(av+s+t) + \gamma A^2}{8t} \left[v(1-a) - s - t - \frac{\gamma A^2}{2}\right] \end{aligned}$$

The derivatives

(i) The partial derivative of CS_L with respect to A is:

$$\begin{aligned} \frac{\partial CS_L}{\partial A} &= -\frac{(2t + 2(av+s) + \gamma A^2)\gamma A}{32t} + \frac{\gamma A}{4t} \left(v - \frac{av+s+t}{2} - \frac{\gamma A^2}{4}\right) \\ &\quad - \frac{\gamma A}{2} \frac{2t + 2(av+s) + \gamma A^2}{8t} \\ &= \frac{\gamma A(8v - 10t - 10(av+s) - 5\gamma A^2)}{32t} \end{aligned} \tag{A-9}$$

(ii) Next, the partial derivative of CS_P with respect to A is:

$$\begin{aligned}
\frac{\partial CS_P}{\partial A} &= -\gamma A - \left(\frac{\gamma A(2(av + s + t) + \gamma A^2)}{32t} - \frac{\gamma A(2(av + s + t) + \gamma A^2)}{8t} + \right. \\
&\quad \left. \frac{\gamma A(v(1 - a) - s - t - \frac{\gamma A^2}{2})}{4t} \right) \\
&= \frac{\gamma A(7A^2\gamma + 14(av + s) - 18t - 8v)}{32t}
\end{aligned} \tag{A-10}$$

Combining Equation (A-9) and (A-10), then:

$$\begin{aligned}
\frac{\partial CS}{\partial A} &= \frac{\gamma A(8v - 10t - 10(av + s) - 5\gamma A^2)}{32t} + \frac{\gamma A(7A^2\gamma + 14(av + s) - 18t - 8v)}{32t} \\
&= \frac{\gamma A(\gamma A^2 - 14t + 2(av + s))}{16t}
\end{aligned} \tag{A-11}$$

Kuhn Tucker conditions of subsection 5.2.1

Using the Consumer Surplus derived in the previous equations, the Kuhn-Tucker necessary conditions for a maximum are:

$$\begin{aligned}
i) \mathcal{L}_A &= \frac{\gamma A(A^2\gamma - 14t + 2(av + s))}{16t} + \lambda r \frac{6t - 2(av + s) - 3\gamma A^2}{8t} \leq 0 \text{ for } A \geq 0 \\
ii) \Pi_P &= \frac{6t - 2(av + s) - \gamma A^2}{8t} Ar - K \geq 0 \\
iii) \lambda \Pi_P &= 0 \quad \lambda \geq 0
\end{aligned} \tag{A-12}$$

Suppose $\lambda = 0$, then the problem shrinks to an unconstrained optimisation as discussed by Proposition 3, which is discussed below. Suppose that $\lambda > 0$, to have $\Pi_P = 0$ (ii), it must be true that the demand for the pirate platform is positive such that $\frac{6t - 2(av + s) - \gamma A^2}{8t} > 0 \implies 2(3t - (av + s)) > \gamma A^2$. By straightforward manipulation and using $0 < \alpha < 1$ as scale parameter ensuring that conditions (i-ii) are satisfied, the positive root is $A = \frac{\sqrt{2\gamma\alpha(3t - (av + s))}}{\gamma}$. Note that α assumes the same role as of β in the previous section. As before, we do not solve explicitly for A .

Proof of Proposition 4

Compare demand pre- and post- ad-blocker adoption:

$$\begin{aligned} D_L^F - D_L^{F-BL} &= \frac{av+s}{6t} + \frac{1}{2} - \frac{t(1+3\alpha) + (av+s)(1-\alpha)}{4t} \\ &= \frac{3t(1-\alpha) - (av+s)(1-3\alpha)}{12t} \end{aligned} \quad (\text{A-13})$$

Suppose that $D_L^F \leq D_L^{F-BL}$, then it must be the case that $3t(1-\alpha) \leq (av+s)(1-3\alpha)$. As $t > \frac{av+s}{3}$ to make A^F real, it should be true that $\frac{3(av+s)(1-\alpha)}{3} < (av+s)(1-3\alpha) \implies 1-\alpha < (1-3\alpha)$, which is clearly false. By contradiction $D_L^F > D_L^{F-BL}$. Next, compare prices pre- and post- ad-blocker adoption:

$$\begin{aligned} p_L^F - p_L^{F-BL} &= \frac{av+s}{3} + t - \frac{t(1+3\alpha) + (av+s)(1-\alpha)}{2} \\ &= \frac{3t(1-3\alpha) - (av+s)(1-3\alpha)}{6} = \frac{(1-3\alpha)(3t - (av+s))}{6} \end{aligned} \quad (\text{A-14})$$

As $3t - (av+s) > 0$, the difference is positive if and only if $1-3\alpha > 0$. Therefore, $p_L^F > p_L^{F-BL}$ as long as $\alpha < \frac{1}{3}$ while $p_L^F < p_L^{F-BL}$ for $\alpha > \frac{1}{3}$. To demonstrate that ad-blockers effectively cut advertisements, compare A^F and A^{F-BL} , then:

$$\begin{aligned} A^F \leq A^{F-BL} &\implies \frac{\sqrt{6\gamma(3t-(av+s))}}{3\gamma} \leq \frac{\sqrt{2\gamma\alpha(3t-(av+s))}}{\gamma} \\ f(A^F) \leq f(A^{F-BL}) &\implies 3t - (av+s) \leq 3\alpha(3t - (av+s)) \implies \alpha \geq \frac{1}{3} \end{aligned} \quad (\text{A-15})$$

By contradiction, for any $\alpha \leq \frac{1}{3}$, advertisements are limited under ad-blocker adoption, whereas for any $\alpha > \frac{1}{3}$, consumers never adopt it. Using results of prices, for any $\alpha \leq \frac{1}{3}$, the LP price is lower than in the pre-adoption case.

Moreover, compare profits when the platform moves second:

$$\begin{aligned} \Pi_P^F - \Pi_P^{F-BL} &= \left[\frac{(3t+av+s)^2}{18t} - F \right] - \frac{(t(1+3\alpha) + (av+s)(1-\alpha))^2}{8t} + F \\ &= \frac{1}{72t} [27t^2(1-3\alpha^2-2\alpha) + (av+s)^2(-5-9\beta^2+18\alpha) + \\ &\quad 6t(av+s)(1-6\alpha+9\alpha^2)] \end{aligned} \quad (\text{A-16})$$

Then, suppose $\Pi_P^F - \Pi_P^{F-BL} \leq 0$ and as $t > av+s$ (Assumption 1), it implies that $27(av+s)^2(1-3\alpha^2-2\alpha) + 6(av+s)^2(1-6\alpha+9\alpha^2) < 27t^2(1-3\alpha^2-2\alpha) + 6t(av+s)(1-$

$6\alpha + 9\alpha^2) \leq (av + s)^2(5 + 9\alpha^2 - 18\alpha)$. Thus, simplifying, it yields $4(7 - 9\alpha^2 - 18\alpha) < 0$, which is true if and only if $\alpha > \frac{1}{3}$. For $\alpha = \frac{1}{3}$ both profits are equal as there is no difference between pre- and post-adoption regarding the advertisement level, whereas for $\beta < \frac{1}{3}$ post-adoption profits are lowered. As $\alpha \in (0, \frac{1}{3})$ to make ad-blockers fully working, then $\Pi_P^F - \Pi_P^{F-BL} > 0$. ■

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