Strategic obfuscation or disclosure by a monopoly when today's substitutes are tomorrow's products

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Abstract

A monopoly decides either to announce or to obfuscate its future products. The monopoly prefers to obfuscate when it controls the costs incurred by consumers to return. This results from the combination of two economic forces. First, hiding information creates consumers with homogeneous expectations. Second, the introduction of costs of returning reduces the waiting value for all consumers simultaneously. Separately, these forces are insufficient to generate higher profit under obfuscation than under revelation. However, together they turn consumers into myopic agents from which the monopoly can extract higher revenue, through intertemporal price discrimination.

Keywords. Multiproduct-firm, monopoly, obfuscation, intertemporal price discrimination

JEL Classification. L12 D21 D42 D83

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1 Introduction

In September 2009, The Economist shed light on the emergence of a new business model un the e-commerce, the *Online Exclusive Sales Market*, at the expense of traditional luxury retailers. This new flourishing market consists of websites (e.g. Venteprivee.com, HauteLook, Rue La La, Net-A-Porter. . .) selling high quality products, mostly in the ready-to-wear industry, to registered consumers. In barely ten years, the world leader, Vente-Privee.com, almost reached 2 billions euro of sales revenues in 2015. Empirical studies of such websites are rare, but one can be found in Helmers et al. (2015). One specificity of this market is the very short duration of product availability (between one day and one week), with a very high turnover. Every day, new sales are opened and old ones are closed, whatever the remaining stock. This contributes, as stated by The Economist, to "*make shopping an urgent and competitive daily activity*". Even though the planning of sales is determined weeks in advance, consumers only discovers the new products at the opening date of the sales. Vanessa Friedman, in the New York Times in 2014, rephrases this idea as "*Get it cheap now before it disappears!*" Another specificity is that shopping on these websites is definitively time-consuming, as it requires consumers to log in everyday. Moreover, some of these platforms manipulate the costs of coming back in the future through different instruments: sales' opening hours, personalized alerts for specific brands or mobile application. Interestingly, VentePrivee.com, the first-mover of this industry, adopted this specific business model before the entry of its competitors, suggesting that it is profitable even without competitors.

This paper provides a rationale for the choice of this business model, using two key elements: consumers' information about their future preference and costs of returning to the seller. I show how a monopoly can use these complementary tools to minimize the level of competition with itself in a dynamic framework. The baseline model developed here is a two period game where one product is sold in each period. Consumers have a unit demand and have to choose between the two goods. The seller manipulate at will the costs incurred by consumers of returning in the second period. To maximize its profit the platform controls these costs and has two possible strategies: (i) disclosure: announce all the products in the first period, so that consumers can choose the product yielding the higher surplus or (ii) obfuscation: hide the second period's good so that consumers only have expectations of the future product. The choice of the information strategy —obfuscation or disclosure— and the level of the costs of returning are two instruments controlled by the seller. Having two different strategic variables separates the role played by each. It is then possible to study how both interact at equilibrium. This double variable choice is especially appropriate to describe situations where firms can only partially manipulate search costs, but still commit on an information strategy, as in the *Online Exclusive Sale Market*.

When there is no intertemporal correlation between the present and future valuations of the products, obfuscation is always profitable. An intuition for this result is that the introduction of optimal costs of coming back, joined with obfuscation, enables the monopoly to turn rational and patient consumers into myopic agents. Indeed, consumers share the same expectations of the future. Therefore they similarly value the option to wait for the next period. By controlling the costs of coming back, the monopoly can reduce the value of this option to wait to zero. Thus the monopoly convinces consumers to completely disregard the future. Because of this myopia, the monopoly can easily price discriminate between the two periods.

The introduction of correlation between the two valuations softens this result. Indeed, today's valuation is a signal on tomorrow's valuation. Because of this correlation, obfuscation cannot totally smooth consumer's heterogeneity. The waiting option is valuated to zero for some but not for all consumers. Correlation reduces the ability of the monopoly to simultaneously reduce the option value to wait without affecting the second period's demand. Above a given threshold of correlation, obfuscation is no longer optimal and it is then better to reveal.

This paper contributes to the growing literature of firm's marketing strategies, such as rebates, behavioral based price discrimination or dynamic pricing. Many markets, such as hotels or restaurants, are using exploiding offers, especially on the Internet. This provides a credible rationale for obfuscation of future products, based on costs of coming back and on intertemporal price differentiation. A contrapositive statement of my result is to claim that industries without return costs or where it is impossible to price differentiate should reveal all the information they have to the consumers. One may find here an additional justification of the full disclosure of information in other markets such as the movie industry.

The question of optimal intertemporal pricing decisions of a monopoly has been extensively studied since Coase (1972)'s durable good seminal paper. A limit case of my modelization, with perfect correlation of valuations, reproduces Coase's result: the monopoly is cannot gain more than a static one period monopoly. Unlike this field of literature, my point of interest is the choice of the information strategy by the monopoly, disclosure or obfuscation.

Most of the theoretical literature about obfuscation has focused on competition. Some models, *à la* Stahl (1989) are models of incomplete search. A proportion of consumers are without search costs. This heterogeneity generates some randomization within the pricing strategy of the firms. In this context, obfuscation is interpreted as the proportion of consumers with search costs. Thus, increasing obfuscation is directly related to softening the competition, as firms compete only for the consumers without search costs. Wilson (2010) modifies Stahl (1989). Wilson considers obfuscation as a modification of the search costs. Another way of softening competition is Ellison and Wolitzky (2012). With the crucial assumption of consumers convex search costs, the introduction of search costs increases the marginal cost of future searches and therefore reduces the competition. Armstrong and Zhou (2011) develop a setting in which firms can generate search frictions to be prominent. Other models assumes a differentiated sophistication between the consumers as in Ellison (2003) or Gabaix et al. (2006). This heterogeneity in degree of consumer rationality can be exploited with, for example, *add-on*. In all this literature, obfuscation allows firms to reduce competition.

Other papers have tried to justify obfuscation for a monopoly. One approach is to use obfuscation to reduce costs. Shin (2005) or Taylor (2014) show that obfuscation helps a monopoly differentiate between high and low valuated consumers. Therefore the monopoly can better allocate its selling efforts. Another approach is to use sequential search models. Most closely related are Petrikaite (2016) and Gamp (2016). Both model a static multi-product monopoly setting prices and search costs. They show that obfuscation can be profitable, through the minimization of sales externalities and that the total effect on the welfare is ambiguous. In contrast, my modelization is fundamentally dynamic, with one product per period, when they have multiple products and only one period. Instead of search costs, I introduce return costs r as there is no possible recall. Finally, I analyze the influence of correlation between valuations, which has been mainly ignored in the previous literature.

The rest of this paper is structured in the following way. Section 2 describes the basic assumptions of the model. I solve it without correlation in Section 3. Section 4 introduces correlation between the valuations of the goods. Concluding remarks are provided in Section 5.

2 Model

A mass 1 of consumers has unit demand for one of two goods $i \in \{1, 2\}$ sold by a monopoly. Consumers have a personal distinct independent[1] valuation $v_i \in [0, 1]$ for each good, drawn from a distribution with cdf F_i and pdf f_i. These valuations are private information, whereas the distributions are common knowledge. Distributions F_i satisfy classical assumption of strictly decreasing inverse hazard ratio,[2] to ensure the concavity of the profits functions. Consumers discount the future with a factor δ and incurs return costs r of returning to the seller. These costs are manipulated at will by the monopoly.[3] The choice of $r \in [0, \infty)$ is costless.

The timing of the game is the following :

$t = 0$ The monopoly chooses whether to reveal or to disclose.

Disclose

$t = 1$ The monopoly chooses p_1 and r. Consumers then discover v_1, v_2, p_1 and r. They choose whether they buy good 1 and whether they come back

$t = 2$ The monopoly chooses a price p_2. Consumers find p_2 and incur r. They choose whether they buy good 2.

Obfuscate

$t = 1$ The monopoly chooses p_1 and r. Consumers then find v_1, p_1 and r. They choose whether they buy good 1 and whether they come back

$t = 2$ The monopoly chooses a price p_2. Consumers find p_2, v_2 and incur r. They choose whether they buy good 2.

It is impossible for consumers to buy good 1 in period 2. The monopoly cannot credibly commit on a second period price.[4] In each situation, disclosure or obfuscation, the monopoly can freely choose three instruments: the prices p_1, p_2 and the return costs r. For clarity, the seller is assumed not to discount the future.

[1] Section 4.1 and 4.2 analyze the presence of positive or negative correlation

[2] This assumption states that $(1 - F_i)/f_i$ is a strictly decreasing function. It is satisfied for a large range of distributions, including the uniform.

[3] Section A.1 analyzes the case of exogenous r.

[4] Section A.2 in the appendix shows that this lack of commitment doesn't affect the equilibrium.

3 Resolution

This section explains the driving forces of the model and establishes a set of normative results between obfuscation and revelation.

3.1 Disclosure

If the monopoly reveals the second period product, the choice of r is straightforward, according to Lemma 1.

Lemma 1. *When the second period good is disclosed, the optimal return cost for the monopoly is* 0*.*

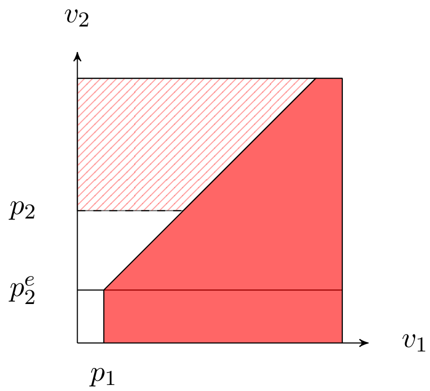
Proof. By contradiction. Let's assume that the monopoly set $r > 0$. Consumers willing to return at the second period are the ones such that: $\delta(v_2 - p_2^e - r) > \max(0, (v_1 - p_1))$, where p_2^e is the expected second period price. Say differently, the second period valuation v_2 is above $p_2^e + r$ for all consumers returning in the second period. In this context, the monopoly's incentives are to set its second period price to $p_2 = p_2^e + r$. Without price commitment, there is no price p_2 simultaneously satisfying the consistency of consumer's expectations, the maximization of the second period's profit and the existence of a second period demand. □

The Figure 1 plots the first and second period demands on all the possible pair of valuations. These demands depends on three parameters: p_1, p_2 and p_2^e, where p_2^e is the expected second period price. The oblique line $v_1 - p_1 = \delta(v_2 - p_2^e)$ represents the indifference curve between buying now or waiting. Consumers whose valuation v_1 is equal to $p_1 + \delta(v_2 - p_2^e)$ are indifferent between purchasing today and coming back in the second period. Therefore, consumers willing to purchase in the second period if and only if : $v_2 \geq p_2^e$ and $v_1 \leq p_1 + \delta(v_2 - p_2^e)$. For a given $v_2 \geq p_2^e$, $F_1(p_1 + \delta(v_2 - p_2^e))$ represents the mass of consumers coming back in the second period. The second period demand function, represented by the dashed area in Figure 1 writes:

$$D_2(p_2, p_2^e, p_1) = \int_{\max(p_2, p_2^e)}^{1} F_1(p_1 + \delta(v_2 - p_2^e)) f_2(v_2)\, \mathrm{d}v_2 \qquad (1)$$

The clients purchasing are the ones coming back and whose second-period valuation is greater than p_2. Thus, v_2 must be greater than $\max(p_2, p_2^e)$. The monopoly

Figure 1: Demands in the revelation case



chooses the second price p_2 maximizing its profit, considering p_2^e fixed. In equilibrium, consumers' expectations are nevertheless consistent. The first order condition of the second period profit, joined with the consistency of the expectations yields the following equation:

$$F_1(p_1)p_2f_2(p_2) = F_1(p_1)(1 - F_2(p_2)) + \int_{p_2}^{1} (F_1(p_1 + \delta(v_2 - p_2)) - F_1(p_1))\, f_2(v_2)\, \mathrm{d}v_2 \tag{2}$$

Section B.1 in the appendix proves that this implicit equation defines a unique second period equilibrium price $p_2^{\mathcal{D}}(p_1) = p_2^{\mathcal{D}}$. The first term of equation 2 is very standard and corresponds to the mass of clients with $v_1 < p_1$ purchasing the second good. For them, the relevant alternative is not to purchase. Alone, it would require the second period price to be equal to the static monopoly price.

The second term of the equation corresponds to the mass of clients arbitrating between the two goods in favor of the second. These arbitrageurs are such that $v_1 > p_1$ but prefer to wait and buy the second good. At a given p_1, the probability to come back in the second period is an increasing function of v_2. Moreover, only clients with a valuation above p_2 choose to come back. These two elements create an incentive to increase $p_2^{\mathcal{D}}$ above the static monopoly price. The mass of arbitrageurs is directly related to the time discount factor δ. In the extreme case of myopic consumers, there wouldn't be any.

Equation 2 thus defines a second period price above the static monopoly price under disclosure, because of consumers arbitrating.

With Figure 1, the monopoly's total profit is:

$$\Pi^{\mathcal{D}}(p_1,p_2) = p_1 \int_{p_1}^{1} F_2(p_2^{\mathcal{D}} + \frac{v-p_1}{\delta}) f_1(v)\,\mathrm{d}v + p_2^{\mathcal{D}} \int_{p_2^{\mathcal{D}}}^{1} F_1(p_1 + \delta(v - p_2^{\mathcal{D}})) f_2(v)\,\mathrm{d}v$$

Which can be rewritten as:

$$\begin{aligned}\Pi^{\mathcal{D}}(p_1,p_2^{\mathcal{D}}(p_1)) =& (p_1 - c_1(p_1,p_2^{\mathcal{D}}))(1-F_1(p_1)) + (p_2^{\mathcal{D}} - c_2(p_1,p_2^{\mathcal{D}}))(1-F_2(p_2^{\mathcal{D}}))\\ &\text{with } c_1(\underset{+}{p_1},p_2) = p_2 \int_{p_2}^{1} \frac{1-F_1(p_1+\delta(v-p_2))}{1-F_1(p_1)} f_2(v)\,\mathrm{d}v\\ &\text{with } c_2(p_1,\underset{+}{p_2}) = p_1 \int_{p_1}^{1} \frac{1-F_2(p_2+\frac{v-p_1}{\delta})}{1-F_2(p_2)} f_1(v)\,\mathrm{d}v\end{aligned}$$

Where c_1 and c_2 can be seen as costs of arbitrage. When the first period price increases, some consumers substitute good 1 for good 2. This leads to a bigger loss with respect to two non substitutable goods. As $c_1(p_1,p_2)$ and $c_2(p_1,p_2)$ are positive functions, the two periods prices are strictly above their static monopoly prices.

The monopoly chooses the optimal first period price maximizing $\Pi^{\mathcal{R}}(p_1,p_2^{\mathcal{D}}(p_1))$. As the analytical definition of this first price in the general case is untractable, Sections 3.3 and 3.4 rely on additional assumptions.

3.2 Obfuscating

The other monopoly's strategy is to obfuscate v_2 in the first period. Clients no longer know v_2. Depending on the return costs r, they might be a second period demand.

Consumers coming back in the second period are those expecting the surplus of waiting to be bigger than the first period surplus. Mathematically:

$$\begin{aligned}&\max(v_1-p_1,0) \le \delta \int_{p_2^e}^{1} (v_2-p_2^e) f_2(v_2)\,\mathrm{d}v_2 - \delta r\\ \Leftrightarrow &\begin{cases} r \le \int_{p_2^e}^{1} (v_2-p_2^e) f_2(v_2)\,\mathrm{d}v_2 \\ v_1 < p_1 + \delta\left(\int_{p_2^e}^{1} (v_2-p_2^e) f_2(v_2)\,\mathrm{d}v_2 - r\right)\end{cases}\end{aligned}$$

This last equation gives the following proposition.

Proposition 1. *Under obfuscation, there exists a maximum value of return costs r compatible with the existence of a second period's demand, defined by a participation constraint.*

$$\int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2)\, \mathrm{d}v_2 \geq r \tag{PC}$$

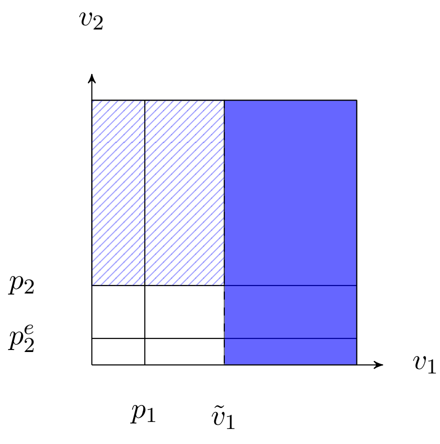
Indeed, if r is too high, there is no demand in the second period because all consumers expect to have a negative surplus of waiting.

Clients coming back in the second period are the ones with a relatively low first period valuation : v_1 must be lower than a given threshold, $\tilde{v}_1$, given by the RHS of the following inequation.

$$v_1 < p_1 + \delta \left(\int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2)\, \mathrm{d}v_2 - r \right) = \tilde{v}_1$$

Where, $\tilde{v}_1$ depends on p_1 and on r only through $(p_1 - \delta r)$. Using (PC), it is straightforward that $\tilde{v}_1 \geq p_1$. Based on these equations, the Figure 2 represents the different demands. The filled area corresponds to the clients not coming back as $v_1 \geq \tilde{v}_1$ and buying in the first period. The hatched area corresponds to the second period demand.

Figure 2: Demands in the obfuscation case



The second period profit of the monopoly in the obfuscation case can be written as:

$$\Pi_2 = F_1(\tilde{v}_1) p_2 (1 - F_2(p_2))$$

This equation immediately states that the optimal second period price is the static monopoly price p_2^m, as $\tilde{v}_1$ does not depends on p_2 but on p_2^e only. In the second period, it is optimal for the monopoly to price as a static monopolist. Indeed, the fact that consumers come back provides no information at all about their possible valuation v_2. Obfuscation of the second period good enables the seller to dissociate the second period profit maximization from the first period choice. Finally, the total profit without correlation can be rewritten:

$$\Pi^{\mathcal{O}}(p_1, r) = p_1(1 - F_1(\tilde{v}_1)) + F_1(\tilde{v}_1)p_2^m(1 - F_2(p_2^m))$$

Simple observation of this profit function $\Pi^{\mathcal{O}}(p_1, r)$ shows that it always profitable to increase p_1, keeping $\tilde{v}_1$ constant. This gives Proposition 2.

Proposition 2. *Under obfuscation, it is optimal for the monopoly to bind the participation constraint of the consumers, defined by (PC):*

$$r = \int_{p_2^m}^{1} (v_2 - p_2^m) f_2(v_2)\, \mathrm{d}v_2$$

This choice of r minimizes the option value to wait for consumers without affecting the second period's demand. A direct consequence of this Proposition is that: $\tilde{v}_1 = p_1$. Return costs reduce entirely the value to wait without affecting the existence of a second period's demand. The profit becomes:

$$\Pi^{\mathcal{O}}(p_1, p_2^m) = p_1(1 - F_1(p_1)) + F_1(p_1)p_2^m(1 - F_2(p_2^m))$$

The maximization with respect to p_1 gives the following equation, ensuring both existence and uniqueness of the equilibrium:

$$p_1 = \frac{1 - F_1(p_1)}{f_1(p_1)} + p_2^m(1 - F_2(p_2^m))$$

Proposition 3. *The full resolution of the model under obfuscation gives the following results :*

$$p_1 = \frac{1 - F_1(p_1)}{f_1(p_1)} + p_2^m(1 - F_2(p_2^m))$$
$$p_2 = p_2^m$$
$$r = \int_{p_2^m}^{1} (v_2 - p_2^m) f_2(v_2)\, \mathrm{d}v_2$$

Interestingly, the discount factor of the consumers δ plays no role at all in the prices, nor in the total profit function. This result drives my interpretation of the mechanism of obfuscation. The optimal choice of return costs forces consumers to completely disregard the future and act as myopic agents. Indeed, on expectation, this future yield a zero surplus.

3.3 Comparisons of the profits

The lack of closed form solutions in the general case imposes here additional restrictions to make analytical comparisons. We assume that the distributions functions are identical : $F_1 = F_2 = F$, and that consumers are infinitely patient : $\delta = 1$.

Under these assumptions, without correlation, there exists[5] a symmetric equilibrium price $\tilde{p}$ in the disclosure case yielding the following total profit :

$$\Pi^{\mathcal{D}} = \tilde{p}(1 - F^2(\tilde{p}))$$

The total profit of the monopoly under disclosure is greater than the static monopoly price. Profits under obfuscation or revelation are compared in Proposition 4:

Proposition 4. *If the two distributions are identical and if consumers are infinitely patient, without correlation, it is always profitable to obfuscate.*

Proof. By definition of $\Pi^{\mathcal{O}}$:

$$\begin{aligned} \Pi^{\mathcal{O}} &= \max_{p_1} p_1(1 - F(p_1)) + F(p_1)\Pi^m \\ &= \max_{(p_1,p_2)} p_1(1 - F(p_1)) + F(p_1)p_2(1 - F(p_2)) \end{aligned}$$

Rewriting $\Pi^{\mathcal{D}}$

$$\begin{aligned} \Pi^{\mathcal{D}} &= \max_{p} p(1 - F^2(p)) \\ &= \max_{p} p(1 - F(p)) + F(p)p(1 - F(p)) \end{aligned}$$

Under disclosure, the monopoly has only one instrument p to maximize its profit, while he has two (p_1 and p_2) under obfuscation. Because of this additionnal constraint, profit under revelation is smaller. □

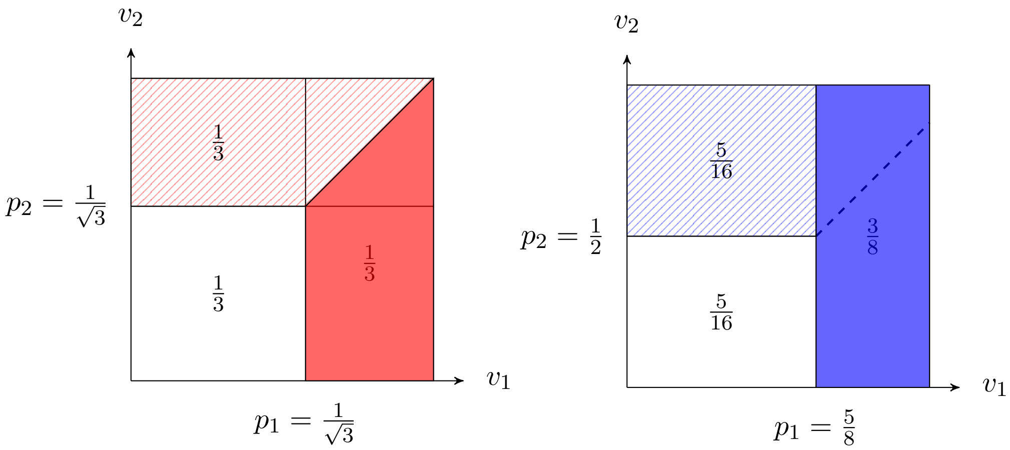
[5]Full proof is in appendix, the idea is to notice that, with these assumptions, the profit is a symmetric function

Under disclosure, the monopoly chooses the same prices $p_1 = p_2 = \tilde{p}$ to prevent consumers from arbitrating. Thus, he only have one available price to maximize its profit. On the other hand, under obfuscation, the monopoly manages to disconnect the second period profit maximization from the first one, and price discriminate between consumers. The following subsection provides an illustrative interpretation of this Proposition, in the case of the uniform distribution.

3.4 Application in the uniform case

This subsection assumes that $F_1 = F_2 = \mathcal{U}_{[0,1]}$ and $\delta = 1$. In this context, demand functions are confounded with areas on Figures 2 and 1. Furthermore, equilibria can be computed analytically. The first period demand is represented in the filled area, and the second period demand in the dashed area of Figure 3.

Figure 3: Equilibria under obfuscation or disclosure



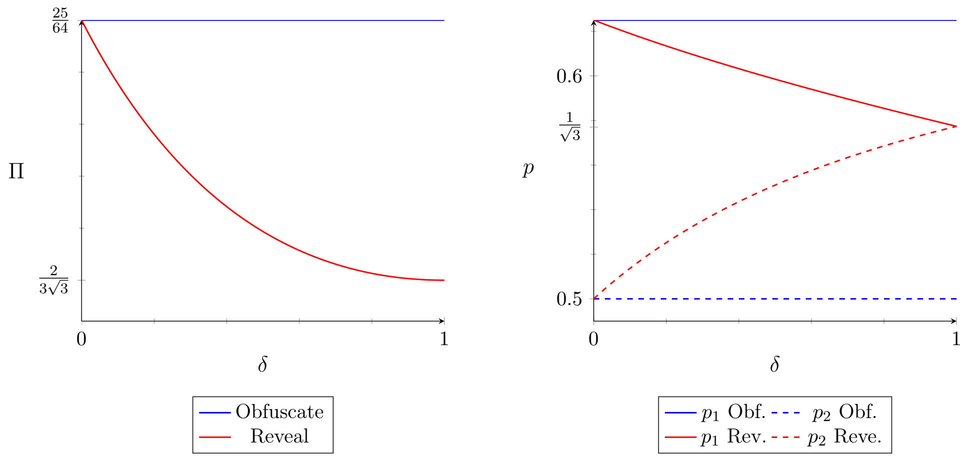
(a) Disclose (b) Obfuscate

When clients are informed, it is optimal to choose the same price for the two periods : $p_1 = p_2 = \dfrac{1}{\sqrt{3}}$. This choice splits in three equal parts the clients purchasing the first good, the second or none. It is not possible for the monopoly to price discriminate, because consumers would then have the possibility to arbitrage. Keeping total demand constant, if the two prices are not equal, more than half of the clients buy the low price good.

With obfuscation, it is possible to price-discriminate. As the clients have no expected surplus of coming back in the second period, they act today as if there were no future. The ones with $v_1 \geq p_1$ buy now, while the ones with low valuations come back in the second period and are priced at the monopoly price $p_2 = \frac{1}{2}$. If the monopoly keep the same prices and reveals v_2, many clients — all the clients in the filled blue area and above the dashed line of Figure 3b — substitute the first (and expensive) good with the second (and cheap) good.

The Figure 4 compares analytically the profits and the prices of obfuscation and of revelation allowing δ to vary from 0 to 1. As already pointed out, δ has no

Figure 4: Comparative statics



(a) Comparison of Profits

(b) Comparison of Prices

influence on the equilibrium prices and profit of the obfuscation case.

The profit of the monopoly is always greater or equal when it obfuscate, compared to revealing. Only for $\delta = 0$, both profits are equal. Indeed, when δ converges to 0, the indifference curve between buying today and tomorrow goes steeper and steeper, and the two demands are very similar. From the point of view of the consumer, when the future doesn't matter anymore ($\delta \to 0$), it is useless to know or not the next valuation. The prices are always more dispersed with obfuscation than without (the red lines are between the blue ones).

A natural interpretation of obfuscation is to turn clients into myopic consumers as they act *as if* $\delta = 0$, even if it is not the case. This result may have very large consequences in term of estimation of δ in environments where there is some return costs.

4 Correlated valuations

This section covers some natural extensions to prove the robustness of the results in more complex settings. Unless otherwise stated, and to ensure tractability, it is assumed that the distributions are identical and uniformly distributed: $F_1 = F_2 = F = \mathcal{U}_{[0,1]}$ and that consumers are infinitely patient: $\delta = 1$.

4.1 Positive correlation

This extension of the base model introduces an arbitrary correlation between valuations v_1 and v_2. To ensure tractability, correlation is introduced through a parameter $\mu \in [0, 1]$, such that:

$$\begin{cases} v_2 = v_1 \text{ with probability } \mu \\ v_2 \sim F_2 \text{ with probability } 1 - \mu \end{cases}$$

We note that, in this context, the probability to have a valuation v_2 below a given threshold is given by:

$$\mathbb{P}[v_2 \leq y] = \mu \mathbb{1}\{v_1 \leq y\} + (1 - \mu) F_2(y)$$

The introduction of correlation has two consequences. First it modifies the repartition of valuation pairs within the square $[0, 1] \times [0, 1]$. There is now a mass μ on the diagonal, while the remaining mass of consumers $(1 - \mu)$ is with $v_1 \neq v_2$. This mass on the diagonal tends to lower the expected maximum valuation of the goods $\mathbb{E}[\max(v_1, v_2)]$. Correlation thus decreases the benefits for consumers and for the seller of having multiple periods. Second, correlation of valuations introduces a possible informational gain for consumers. The first period's valuation is a signal on the second period's valuation, allowing consumers to form heterogenous expectations of their future.

The monopoly is assumed not to exclude anyone.[6]

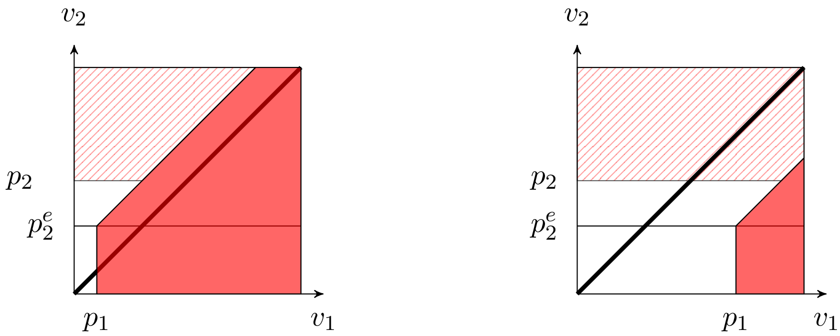
4.1.1 Disclosure

Under disclosure, the correlation plays no informational role as consumers are already privately informed of v_1 and v_2 at the beginning of the gain. Compared

[6] I implicitly assume here that return costs are sufficiently low to clear the entire market after an infinite number of periods. It is indeed impossible for such a monopoly to credibly commit not to clear the market. This assumption is respected if $F_1 = F_2 = \mathcal{U}_{[0,1]}$, but can be violated for other distributions.

to the previous sections, the repartition of the consumers on the (v_1, v_2) square is now slightly different, as there is now a mass of consumers for which both goods are identical. Thus the incentives to have equal prices for the monopoly should be reinforced and the problem remains similar. The seller chooses not to have any return costs. The Figure 5 represents the different demands, depending on p_1, p_2^e and p_2. There are two different regimes in the demands of the first and the second good, as the mass of clients on the diagonal can switch from one good to the other.

Figure 5: Disclosure with $\mu > 0$



(a) $p_1 \leq p_2^e$ (b) $p_1 \geq p_2^e$

The only equilibrium price (p_1, p_2) is such that $p_1 \geq p_2$.

To go further in this resolution, one would need some analytical computations, performed in subsection 4.1.3.

4.1.2 Obfuscation

Under obfuscation, the introduction of correlation both changes the repartition of valuations on the (v_1, v_2) square and plays an informational role.

First, consumers compare their utility for the first good with their expected utility for the second period, to determine their optimal action.

$$U_1 = \begin{cases} 0 & \text{if } v_1 \leq p_1 \\ v_1 - p_1 & \text{otherwise} \end{cases}$$

$$U_2 = \begin{cases} (1-\mu) \int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2)\, \mathrm{d}v_2 - r & \text{if } v_1 \leq p_2^e \\ (1-\mu) \int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2)\, \mathrm{d}v_2 - r + \mu(v_1 - p_2^e) & \text{otherwise} \end{cases}$$

As the monopoly doesn't want to exclude anyone from the market, it is sufficient and necessary not to exclude the clients with the worst signal on their future valuation, i.e. the ones with $v_1 = 0$. This remark leads to Proposition 2′.

Proposition 2′. *Under obfuscation, it is optimal for the monopoly to bind the participation constraint of the consumers, defined by:*

$$r_{\max} = (1-\mu)\int_{p_2^e}^{1}(v_2 - p_2^e) f_2(v_2)\,\mathrm{d}v_2$$

Proof. Let's assume that $r < r_{\max}$. It is possible to increase p_1 and r, such that the first period demand and the second period demand remains unchanged. This double deviation would yield a strictly higher profit. □

Proposition 2′ simply extends Proposition 2. When there is correlation, the monopoly charges the maximum return costs without excluding anyone from the market.

In this setting, demand functions can be rewritten and the first order condition defining the second period price becomes:

$$p_2 f(p_2) = (1 - F(p_2)) - \frac{1}{1 + \frac{1}{\mu(1-F(\frac{p_1-\mu p_2}{1-\mu}))}}$$

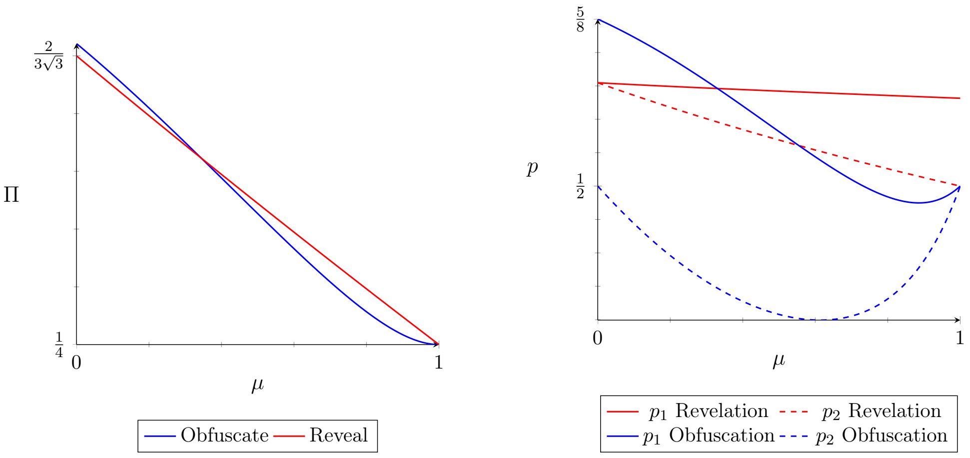
Correlation modifies the second period's price optimisation problem through the second term. This equation establishes the existence and the uniqueness of p_2. It also proves that p_2 is smaller than the static monopoly price at equilibrium. Unfortunately, p_2 is no longer independent of p_1 which prevents us from having a closed form solution for both equilibrium prices.

4.1.3 Comparisons of the profits

Figure 6 provides analytical computations of the profit and the prices of the monopoly in the revelation and obfuscation cases. The first observation regarding the profit is that the correlation decreases the profit in both cases. Indeed, correlation between v_1 and v_2 tends to lower $\max(v_1, v_2)$ and thus, clients have a lower ex-ante valuation for this two period game. The second observation is stated in Proposition 5.

Proposition 5. *With infinitely patient consumers and uniform distributions, there exists a correlation value above which it is not profitable anymore to obfuscate.*

Figure 6: Comparison of Obfuscation and Revelation



(a) Profit (b) Prices

The economic intuition behind this result is simple. Correlation is more harmful to the obfuscation case because it also set a maximal value to the possible return costs. When correlation increases, consumers tends to form more heterogeneous expectations about their future. Return costs are less and less efficient to lower the option value to wait. Therefore, one cannot any longer turn consumers into myopic consumers. Section A.1 establishes that too small search costs is counterproductive for obfuscation.

More generally, obfuscation is efficient as long as it reduces the heterogeneity in the expected surplus of consumers of their future. When correlation is too important, today's valuation is sufficiently informative about tomorrow's valuation. Thus, return costs cannot any longer turn consumers into myopic agents.

4.2 Negative correlation

To complete the model, this section introduces some negative correlation between the valuations in the following way:

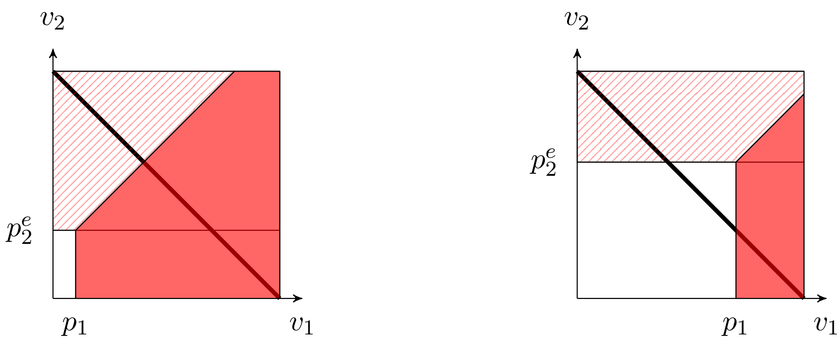
$$\begin{cases} v_2 = 1 - v_1 \text{ with probability } \mu \\ v_2 \sim F_2 \text{ with probability } 1 - \mu \end{cases}$$

Once again, correlation modifies through two channels. First the distribution of pairs of valuations changes. As correlation is negative, the expected surplus of consumers increases. Second consumers are now partially informed about their future valuation and can form heterogeneous expectations.

4.2.1 Revealing

Under revelation, there exists two different demand regimes for the first and the second good, depending on p_1, p_2^e. Figure 7 represents these different demands. Indeed, the diagonal with a mass μ can now either intersect the demand for no good (in white, Figure 7b) or not (Figure 7a).

Figure 7: Negative correlation



(a) No intersection (b) Intersection

In the uniform case, the only equilibrium is such that $p_1 + p_2 \geq 1.$, i.e. Figure 7b. We rely on computational estimation to provide additional results in section 4.2.3.

4.2.2 Obfuscation

Consumers choose to come back or not, depending on their utility in the first period and in their expected utility of waiting:

$$U_1 = \begin{cases} 0 & \text{if } v_1 \leq p_1 \\ v_1 - p_1 & \text{otherwise} \end{cases}$$

$$U_2 = \begin{cases} (1-\mu) \int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2) \, \mathrm{d}v_2 - r & \text{if } v_1 \geq 1 - p_2^e \\ (1-\mu) \int_{p_2^e}^{1} (v_2 - p_2^e) f_2(v_2) \, \mathrm{d}v_2 - r + \mu(1 - v_1 - p_2^e) & \text{otherwise} \end{cases}$$

Return costs have to be sufficiently low not to exclude the clients not purchasing in the first period and with the worst signal on the second period good, i.e. the ones with $v_1 \to_- p_1$. This remark leads to Proposition 2''.

Proposition 2''. *Under obfuscation, it is optimal for the monopoly to bind the participation constraint of the consumers, defined by:*

$$r_{\max} = (1-\mu)\int_{p_2^e}^{1}(v_2 - p_2^e)f_2(v_2)\,\mathrm{d}v_2$$

Proof. Proven with a double deviation argument, increasing simultaneously r and p_1 □

Once again, this result is simply an extension of Proposition 2 in presence of negative correlation.

4.2.3 Comparison of the profits

Using the results from Sections 4.1 and 4.2, the Figure 8 represents the profits under obfuscation and revelation for any negative correlation.

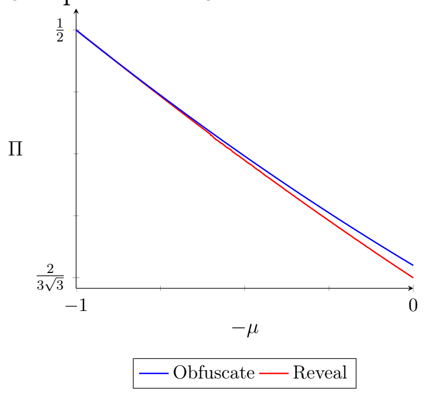
Obfuscation remains the best strategy for all negative correlations. Indeed, high valuated consumers in the first period are low valuated consumers in the second. Thus, it is easier for the monopoly to convince first period consumers with a high valuation to buy now and to generate a myopic behavior. In the same time, low valuated consumers in the first period have higher expectations of their future surplus. Participation constraints are easier to satisfy. Therefore having obfuscation more profitable than revelation is quite meaningful for negative correlations.

As shown by Figure 8, the difference between the two strategies reduces as the correlation converges to -1. Indeed, with a perfect negative signal, it is useless to obfuscate as consumers precisely know their future valuation.

5 Conclusions

This paper developed a model explaining how a seller can benefit from obfuscation, when today's substitutes are tomorrow's products, using two instruments: the information strategy and the level of return costs. The role played by each variable has been extensively detailed: obfuscation forces consumers to have homogeneous expectations about their future surplus, while return costs lower the level of this expected surplus to zero. Thus, consumers behave myopically when they are in fact infinitively patient.

Figure 8: Comparison of Obfuscation and Revelation



The introduction of positive correlation between the goods is more harmful for obfuscation than for revelation. As today's valuation is a signal on tomorrow's expected surplus of the future, return costs cannot any longer reduce totally the value to wait for all consumers simultaneously. As a result, I find that there exists a range of correlation above which obfuscation would be counterproductive compared to revelation.

Using the concept of *flash sales*, the *Online Exclusive Sales* industry is in fact preventing consumers from anticipating the future products that may appears in the following weeks. This strategy based on information obfuscation and costs of waiting is a credible alternative to some criticised practices, such as behaviorial based price discrimination or discretionary rebates used to screen consumers. Regulation authorities might want to interfere to control this kind of schemes, even though the efficiency of the standard tools at their disposal seems to be limited.

Future research may explore some extensions of this model, introducing for example competition or changing the dynamics of the intertemporal modelization. For instance, a flow of consumers coming at each period and a small positive correlation in the valuations of the goods could be sufficient to generate under obfuscation price patterns similar to Sobel (1984).

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A Additional extensions

A.1 Exogeneity of return costs

This section assumes that the return costs $r > 0$ are fixed and cannot be modified by the monopoly. Therefore, the only possible strategic variables are the prices p_1, p_2 and the possibility to obfuscate v_2.

A.1.1 Disclosure

If there exists some strictly positive return costs in the revealing case, the monopoly needs to commit on a second period price[7]. Nevertheless, Section A.2 proves that this price commitment doesn't modify the profit of the monopoly when it disclose in the standard case. In this context, the total profit is written as :

$$\Pi^{\mathcal{D}} = p_1 \int_{p_1}^{1} F_2(p_2 + r + v_1 - p_1) f_1(v_1)\,\mathrm{d}v_1 + p_2 \int_{p_2+r}^{1} F_1(p_1 + v_2 - p_2 - r) f_2(v_2)\,\mathrm{d}v_2$$

As previously observed when r was endogenous, an increase of r is prejudicial to the monopoly. Thus, one can state that $\Pi^{\mathcal{D}}$ is a decreasing function of r.

A.1.2 Obfuscating

The introduction of an exogenous r doesn't modify the second period problem for the monopoly. Clients coming back in the second period are priced at the monopoly price. The participation constraint PC exhibits a maximal search cost above which no client would come back:

$$r_{\max} = \int_{p_2^m}^{1} (v_2 - p_2^m) f_2(v_2)\,\mathrm{d}v_2$$

For $r > r_{\max}$, there is no second period demand, and the model collapses to a single good monopoly. For $r \leq r_{\max}$, the profit of the monopoly can be rewritten as:

$$\begin{aligned}\Pi^{\mathcal{O}}(p_1) =& p_1(1 - F_1(p_1 + \delta(r_{\max} - r)) + p_2^m F_1(p_1 + \delta(r_{\max} - r))(1 - F_2(p_2^m)) \\ =& F_1(p_1 + \delta(r_{\max} - r))(\Pi_2^m - p_1) + p_1\end{aligned}$$

The profit under obfuscation is an increasing function of r, as long as it is below $r_{\max}$.

[7] Proof of this statement is available in Appendix B.3

A.1.3 Comparison of the profits

Using the previous observations, Proposition 6 is immediate:

Proposition 6. *There exists a range of return costs for which it is profitable to obfuscate.*

Proof. $\Pi^{\mathcal{O}}$ is an increasing function of r, and $\Pi^{\mathcal{D}}$ decreasing. For $r < r_{\max}$, both profit functions are continuous, and $\Pi^{\mathcal{D}}(r=0) < \Pi^{\mathcal{D}}(r = r_{\max})$. According to the intermediate value theorem, there exists a range $(\underline{r}, r_{\max}]$, such that the monopoly is strictly better off with obfuscation. □

This results states that the choice of an information strategy (obfuscation or disclosure) can be used to generate leverage on preexisting return costs. Under revelation, return costs are always prejudicial, but obfuscation allows firms to use these costs to increase their profit.

A.2 Commitment power

To better understand the role played by obfuscation itself, it is important to rule out any commitment effect.This section introduces a hypothetical scenario, where the monopoly can disclose the second good and perfectly commit on a second period price p_2 at the beginning of the first period.

Clients no longer have expectations, and they can perfectly choose whether to wait or to buy in the first period. Thus, total profit of the monopoly is:

$$\Pi^{\mathcal{C}\text{omm}}(p_1, p_2) = p_1 \int_{p_1}^{1} F_2(p_2 + \frac{v - p_1}{\delta}) f_1(v)\, \mathrm{d}v + p_2 \int_{p_2}^{1} F_1(p_1 + \delta(v - p_2)) f_2(v)\, \mathrm{d}v$$

This profit is exactly the same as the one without price commitment, excepted that p_2 and p_1 are no longer tied by $p_2^{\mathcal{R}}(p_1)$. The general case doesn't exhibit closed form solution, but with infinitely patient consumers and symmetry of distributions, it is possible to display the same symmetric equilibrium as without price commitment. Indeed the two first order conditions in the symmetric equilibrium are identical and they satisfy the optimality of the second period price without commitment.

Therefore, Result 4 can be extended with with price commitment in the revelation case.

A.3 Similar search costs at the two periods

One may argue it is unrealistic to have high return costs in the second period and no return costs in the first one. Even though this modelization is a standard one in the search literature, it remains difficult for an industry to manipulate the return costs and make them vary from one period to another. If the model is a succession of sales, $\{\ldots, A, B, C, \ldots\}$, return costs of sale B cannot be simultaneously high in the pair $\{A, B\}$ and low in the pair $\{B, C\}$ as they should.

To circumvent this issue, two additional constraints are introduced: return costs have to be identical at the two periods and the total expected surplus of the consumers must be positive to ensure the existence of the market.

When there is no correlation, these constraints combine in the following inequation:

$$(1 - F_1(\tilde{v}_1))(\int_{\tilde{v}_1}^{1} (v_1 - p_1) f_1(v_1)\,\mathrm{d}v_1) + F_1(\tilde{v}_1)(\int_{p_2}^{1} (v_2 - p_2) f_2(v_2)\,\mathrm{d}v_2 - r) \geq r \tag{PC'}$$

With the same notations as previously. In the revelation case, this additional constraint is not binding as optimal return costs were already null. In the obfuscation case, this is no longer true and, depending on F_1 and F_2, the PC' constraint could be binding.

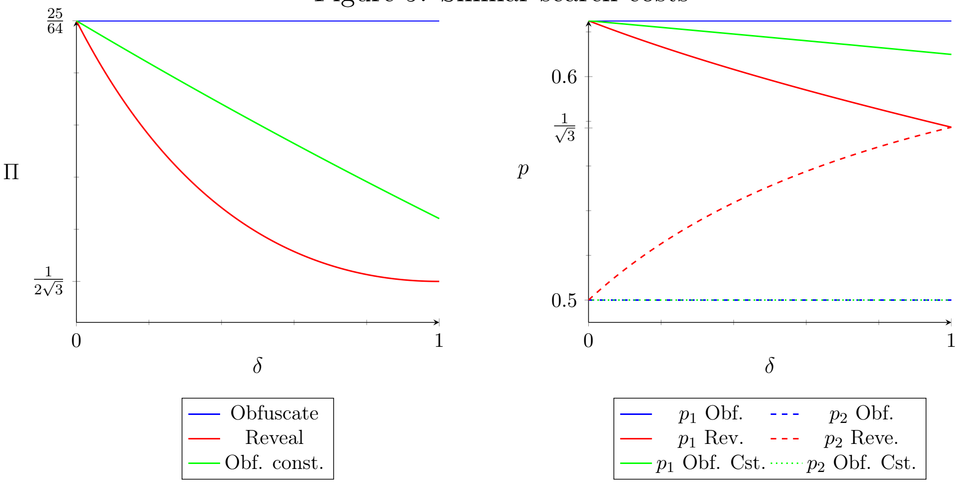
As this additional constraint are computationally complex, numerical computations in the uniform case are provided. In this context, the monopoly limits its return costs to a lower level because PC' is binding. This also impacts the first period price.

The Figure 9 represents the profit and prices of this *constrained obfuscation case* in green. It is still profitable to obfuscate when there are the same return costs in the uniform case. Therefore, there exists a range of parameter values $[0, \hat{\mu}]$ such that, even with these additional constraints, it is still profitable to obfuscate when there is correlation. This range is included in the previous one as obfuscation has been limited.

A.4 Choice of first period product

This section provides a rationale for the optimal choice of product in the first or second period, when there is no correlation between the two valuations and with infinitively patient consumers.

Figure 9: Similar search costs



(a) comparison of Profits

(b) comparison of Prices

Let's assume there is two distributions F_a and F_b, such that F_a and F_b yield the same monopoly profit Π^m. The monopoly has to choose whether it should sell product a or b at period 1, and the other at period 2. Without loss of generality, $p_a^m < p_b^m$, where p_k^m is the monopoly price of distribution k. Distribution F_a is more condensed around the monopoly price, while distribution b is more dispersed.

Under Revelation, with price commitment, the monopoly is indifferent between a and b in the first period. Under Obfuscation, it is better off with the product with the higher variance of valuations in the first period.

The economic intuition is quite straightforward:

- With revelation: Consumers already know their valuations, and arbitrage between the two good. The profit can be written as:

$$\Pi^{\mathcal{R}}(p_1, p_2) = p_1 \int_{p_1}^{1} F_2(p_2 + v - p_1) f_1(v) \, \mathrm{d}v + p_2 \int_{p_2}^{1} F_1(p_1 + v - p_2) f_2(v) \, \mathrm{d}v$$

  If this function admits an optimal vector of prices (p_1, p_2), then inverting F_1 and F_2, the vector (p_2, p_1) would be optimal too. Thus, the monopoly is indifferent between good a or good b in the first period.

- With obfuscation: The profit of the monopoly writes:

$$\begin{aligned}\Pi^{\mathcal{O}}(p_1,p_2) =& p_1(1-F_1(p_1)) + F_1(p_1)\Pi^m \\ =& \Pi^m + p_1(1-F_1(p_1)) - \Pi^m(1-F_1(p_1))\end{aligned}$$

  The monopoly has to choose between : $(F_1 = F_a, F_2 = F_b)$ and $(F_1 = F_b, F_2 = F_a)$. As $p_a^m < p_b^m$ and $p_a^m(1-F_a(p_a^m)) = p_b^m(1-F_b(p_b^m))$, $(1-F_a(p_a^m)) > (1-F_b(p_b^m))$. Thus, distribution F_a is more penalized by the last Π^m term than distribution F_b. It follows that the monopoly will prefer distribution F_b in the first period.

Economically speaking, it is better to have in the first period the good with a lot of variance in its valuation. Indeed, as the monopoly prices above the static monopoly price, a very concentrated distribution around the monopoly price would be very penalized. This result tends to corroborate Petrikaite's mass versus niche product analysis.

A.5 Multiple periods in the uniform case

This extension allows for multiples periods. The monopoly is still facing a mass 1 of consumers willing to buy one in N goods. The monopoly sells good t at period t and has the choice between revealing the whole vector $(v_2, ...v_N)$ at the beginning of the first period or not. It can also choose a search costs s_i, paid by the consumers at the beginning of period i.

A.5.1 Revelation

Focusing once again on the symmetric equilibrium : $p_t = \tilde{p}$, the total profit of the monopoly can be written as: $\Pi^{\mathcal{D}} = \tilde{p}(1-F(\tilde{p})^N)$. Some simple algebra yields: $\tilde{p} = (\frac{1}{N+1})^{\frac{1}{N}}$ and $\Pi^{\mathcal{D}} = \frac{N}{(N+1)^{1+\frac{1}{N}}}$

Under revelation, all periods are identical because of the arbitrage possibilities, and thus the price doesn't depend on t. The total number of periods tends to increase the equilibrium price and the profit of the monopoly converges to one. This result can first look like a counter-example to Coase's conjecture, but the monopoly is selling a *new* product at each period, as opposed to the definition of the durable good monopoly. Nevertheless, relying on Nava and Schiraldi (2016), Coase's conjecture has to be understood in terms of market clearance. From this point of view, when time goes to infinity, the monopoly fulfills all the demand and clears the market.

A.5.2 Obfuscation

The monopoly can achieve higher profits with obfuscation. There exists some value for the return cost at period t such that consumers have no expected surplus of coming back. Indeed, the expected surplus of coming back at time $t+1$ can be written as:

$$ES_{t+1} = \int_{p_{t+1}}^{1} (v - p_{t+1}) f(v) dv - r_{t+1} + \delta ES_{t+2}$$

With the final condition $ES_{N+1} = 0$, there exists a sequence $(r_t)_{t \in \{1,..,N\}}$ such that:

$$r_t = \int_{p_{t+1}}^{1} (v - p_{t+1}) f_{t+1}(v) dv$$

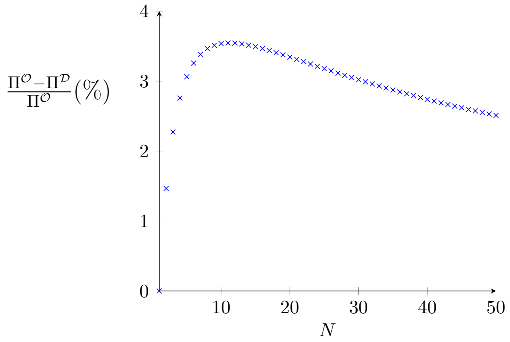
This sequence of search costs ensures that, at any period t, consumers are willing to come back, but expect no surplus of the future. Thus, consumers are perfectly myopic and the optimal prices can be founded by backward induction. It is possible to exhibit an analytical recursive sequence of prices and profit for the obfuscation case, for $t \in \lfloor 0, N \rfloor$.

For a given value of N, the sequence of prices is decreasing when time goes to the final period. More interestingly, the total profit earned by the monopoly when there is N period is increasing in N and converges to 1.

A.5.3 Comparison of the cases

Let's compare the efficiency of these two strategies with the number of periods. The Figure 10 represents the relative gain of obfuscation with respect to disclosure as a function of the total number of periods. When the number of periods is infinite, both modes are able to perfectly price discriminate and extract all the surplus of the consumer, so the difference converges to zero. Obfuscation allows the monopoly to treat the consumer as if they were myopic leading to a more efficient price discrimination and a higher profit. This result remains true, even with multiple periods. Interestingly, there exists an optimal number of periods leading to a maximal gain with obfuscation compared to revelation.

Figure 10: Relative gain of Obfuscation with the number of periods



B Proofs

B.1 Decreasing function

Proof. This subsection proves that $A : x \to \int_x^1 \frac{F_1(p_1 + \delta(v-x))}{f_2(x)} f_2(v)\,\mathrm{d}v$ is a decreasing function.

Using Leibniz's rule :

$$A'(x) = -F_1(p_1) + \int_x^1 \left[\frac{-\delta f_2(x) f_1(p_1 + \delta(v-x))}{f_2^2(x)} - \frac{f_2'(x) F_1(p_1 + \delta(v-x))}{f_2^2(x)}\right] f_2(v)\,\mathrm{d}v$$

Intuitively, the last term of this derivative can be positive. As the $\frac{1-F}{f}$ ratio is decreasing by assumption, f_2' cannot be *too positive*. This observation implies that $\frac{-f'}{f^2} \leq \frac{1}{1-F}$.

Thus :

$$A'(x) \leq -F_1(p_1) - \delta \int_x^1 \frac{f_1(p_1 + \delta(v-x))}{f_2(x)} f_2(v)\,\mathrm{d}v + \underbrace{\int_x^1 \frac{F_1(p_1 + \delta(v-x))}{1 - F_2(x)} f_2(v)\,\mathrm{d}v}_{B(x)}$$

Doing an Integration By Part on this last term gives:

$$
\begin{aligned}
B(x) &= \left[\frac{-(1-F_2(v))F_1(p_1+\delta(v-x))}{1-F_2(x)}\right]_x^1 + \delta \int\limits_x^1 \frac{f_1(p_1+\delta(v-x))(1-F_2(v))}{1-F_2(x)}\,\mathrm{d}v \\
&= F_1(p_1) + \delta \int\limits_x^1 \frac{f_1(p_1+\delta(v-x))(1-F_2(v))}{1-F_2(x)}\,\mathrm{d}v
\end{aligned}
$$

Thus :

$$
A'(x) \leq \frac{\delta}{f_2(x)(1-F_2(x))} \int\limits_x^1 f_1(p_1+\delta(v-x))\big(f_2(x)(1-F_2(v)) - (1-F_2(x))f_2(v)\big)\,\mathrm{d}v
$$

As $v \geq x$ and $\frac{1-F}{f}$ is decreasing, it is possible to conclude that:

$$
f_2(v)(1-F_2(x)) \geq f_2(x)(1-F_2(v))
$$

This last observation establishes that $A'(x) \leq 0$. □

B.2 Existence of a Symmetric Equilibrium without correlation

With infinitely patient consumers and symmetric distributions, the arbitrage costs in $\Pi^{\mathcal{D}}$ are symmetric: $c_1(p_1,p_2) = c_2(p_2,p_1)$. It is then striking that the profit is a symmetric function: $\forall(x,y), \Pi^{\mathcal{D}}(x,y) = \Pi^{\mathcal{D}}(y,x)$. The profit can be then written as a function of p_1 and $h = p_2 - p_1$.

$$
\begin{aligned}
\Pi^{\mathcal{D}} &= p_1 \int\limits_{p_1}^1 F(p_2+v-p_1)f(v)\,\mathrm{d}v + p_2 \int\limits_{p_2}^1 F(p_1+v-p_2)f(v)\,\mathrm{d}v \\
&= p_1 \int\limits_{p_1}^1 F(v+h)f(v)\,\mathrm{d}v + (p_1+h) \int\limits_{p_1+h}^1 F(v-h)f(v)\,\mathrm{d}v
\end{aligned}
$$

And the derivative of the profit are:

$$\frac{\partial \Pi^{\mathcal{R}}}{\partial p_1} = \int_{p_1}^{1} F(v+h)f(v)\,\mathrm{d}v + \int_{p_1+h}^{1} F(v-h)f(v)\,\mathrm{d}v$$
$$- (p_1+h)F(p_1)f(p_1+h) - p_1 F(p_1+h)f(p_1)$$
$$\frac{\partial \Pi^{\mathcal{R}}}{\partial h} = p_1 \int_{p_1}^{1} f(v+h)f(v)\,\mathrm{d}v - (p_1+h) \int_{p_1+h}^{1} f(v-h)f(v)\,\mathrm{d}v$$
$$+ \int_{p_1+h}^{1} F(v-h)f(v)\,\mathrm{d}v - (p_1+h)F(p_1)f(p_1+h)$$

With $h = 0$, both FOC conditions becomes identical and are simply:

$$p_1 F(p_1)f(p_1) = \int_{p_1}^{1} F(v)f(v)\,\mathrm{d}v = \frac{1 - F^2(p_1)}{2}$$

So, the couple (p, h) where p is defined by $2pF(p)f(p) = 1 - F^2(p)$, and $h = 0$ is a solution. Lastly, there exists such a p by monotonicity of the LHS and RHS of this equation.

B.3 Price Commitment with an exogenous s

This subsection proves that, without price commitment, there is no equilibrium in the revealing with exogenous return costs. Clients willing to come back in the second period are the ones for which:

$$v_2 - p_2^e - s \geq v_1 - p_1$$

And the second period profit is simply :

$$\Pi_2 = p_2 \int_{\max(p_2^e + r, p_2)}^{1} F_1(p_1 + v - p_2^e - s) f_2(v_2)\,\mathrm{d}v_2$$

As, at equilibrium, $p_2 = p_2^e$, the following equation holds: $p_2^e + r > p_2$. Thus, the demand doesn't depend on the second period price, and there is no equilibrium price! Simply stated, consumers expect to pay $r + p_2^e$ in the second period. Thus, they only come in the second period if v_2 is strictly greater than $p_2^e + r$. Once they paid r to come back, the monopoly has an incentive to increase its price, and there is no price p_2 satisfying simultaneously the consistency of the beliefs and the optimality conditions.