Prioritization vs zero-rating: Discrimination on the internet

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Abstract

This paper analyzes two business practices on the mobile internet market, paid prioritization and zero-rating. Both violate the principle of net neutrality by discriminating different content types, the former in terms of quality, the latter in terms of price. In recent years these practices have attracted considerable media attention and regulatory interest. The EU, and until recently the US have banned paid prioritization but tolerated zero-rating under conditions. In our model, we assess the welfare effects of different regulatory regimes. When the value of traffic for content providers is limited, the ISP's optimal policy is zero-rating. When the value of traffic is high, the optimal policy is paid prioritization. In this case, the interests of the consumers and the ISP may be aligned.

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1 Introduction

Net neutrality is a regulatory principle that prohibits discrimination on the internet, mandating Internet Service Providers (ISPs) to treat all data packets equally. In this paper, we build a simple model to compare and contrast prioritization and zero-rating, two business practices that violate this principle in terms of the speed and the price of data delivery, respectively. In recent years, both business practices have been widely discussed and debated both in policy circles and the media. However, to the best of our knowledge, ours is the first attempt to investigate the two practices within the same model.

The principle of net neutrality prohibits both **quality** differentiation and **financial** discrimination of different types of content. With quality differentiation, which is usually referred to as (paid) prioritization, the ISP gives priority to some privileged content over the others. In a context of data congestion, the prioritized content has a better quality –faster delivery– while the non-prioritized content types experience congestion in the form of jitters and/or delays. Giving priority to one content over the others is a way for the ISP to differentiate further the different content providers (CPs) by adding another dimension of differentiation, the delivery speed. Obviously, as this can provide an advantage to the prioritized content or, alternatively, a disadvantage to the non-prioritized ones, the content providers may be ready to compensate the ISP for having the priority. A typical example of financial discrimination is mobile data plans with zero-rated content. With such a contract, users subscribe for a package with a monthly data cap but the usage of some content (e.g. Facebook or Netflix) does not count against this data cap, i.e., the ISP charges different rates for different types of content. Clearly, zero-rating is thus another way to create differentiation between content types, financial in this case.

Due to the indisputable importance of the internet, its regulation is a highly contentious issue. Different countries adopt widely different regulatory approaches and even within one jurisdiction, net neutrality regulations tend to be the subject of frequent changes. Moreover, regulators often have a contrasted attitude towards prioritization and zero-rating. For example, in the US, the need to regulate zero-rating arose in 2011 after a complaint about MetroPCS exempting Youtube from it's customers' data cap. The first set of net neutrality rules adopted after this case was the 2015 Open Internet Order. It explicitly mandated a case-by-case treatment of zero-rating, i.e., zero-rating was tolerated, in contrast with paid prioritization which was prohibited by a bright line rule. The FCC, the US telecommunications regulatory body, investigated the four zero-rating offers in place in 2016 and expressed concern over two them (Verizon's FreeBee Data and AT&T's Sponsored Data programs) likely harming consumers[1]. This finding did not have any legal effect as the FCC's composition changed soon after under the Trump administration, and all investigation into zero-rating was stopped. Moreover, the new FCC voted a repeal of the 2015 rules and abolishing net neutrality protections in December 2017. If the repeal is upheld in courts, the lack of rules will result in both business practices being permitted in the US.

In 2015, the EU decided to adopt rules similar to the US regulation that was in place between 2015 and 2017. Indeed, BEREC published Guidelines for net neutrality (BEREC, 2016) mandating a case-by-case treatment of zero-rating offers by national regulatory authorities, while explicitly banning paid prioritization. In practice, according to the Guidelines, some NRAs banned specific zero-rating offerings (see e.g. the case of Telenor in Hungary and Telia in Sweden), while other offerings were found legal (Proximus in Belgium and T-Mobile in Germany). A report by the European Commission (2017) describes most of these cases in detail. While Canada also decided not to ban zero-rating ex-ante, in practice CRTC, its regulatory authority took a very strict approach toward it, de facto prohibiting both zero-rating and paid prioritization[2] . For a detailed summary of the history of zero-rating regulation up to the end of 2016, see Yoo (2016).

The objective of this paper is to compare the two types of business practices, ans thus evaluating the often differentiated policy towards them. More specifically, we construct a model to compare three regimes that could be observed in the mobile internet market:

[1] https://apps.fcc.gov/edocs_public/attachmatch/DOC-342982A1.pdf

[2] https://arstechnica.com/tech-policy/2017/04/as-us-prepares-to-gut-net-neutrality-rules-canada-strengthens-them/

1. Prioritization (P)

Under prioritization, the ISP discriminates between content providers and the content supplied by CP_i has a priority access over the content supplied by CP_j. This means that if there is congestion on the internet, the download quality is not identical for the two types of content. The priority content provider CP_i may or may not compensate the ISP for getting priority. Importantly, consumers pay the same price for accessing both types of content.[3]

2. Zero-rating (ZR)

Under zero-rating, the consumers are not charged the same price by the ISP when they access content of CP_i than when they access content of CP_j. More precisely, the marginal rate charged by the ISP is zero for content i and is positive for content j. In other words, the ISP financially discriminates the two content providers that are identical in terms of download quality. The zero-rated content provider CP_i may or may not compensate the ISP for this service.[4]

3. Net Neutrality (NN)

Under net neutrality, the ISP cannot discriminate between content providers, so this regime can serve as a natural benchmark. In particular, net neutrality prohibits quality differentiation like prioritized content and financial discrimination like applying a lower rate for some types of content. A consequence of net neutrality is the absence of financial transfers between the ISP and the CPs.

In our model, there are two competing content providers. The CPs are two-sided platforms financed by advertising and they compete to attract internet users. They offer differentiated content, we use the Hotelling model to represent horizontal differentiation of content types. Importantly, we suppose asymmetric content providers with one CP (the weak CP) being located at the extreme of the Hotelling segment, while the other (the strong CP) being located in the interior, close to the other extreme. To access content, users must subscribe to an internet service provider, that we suppose

[3]Note that we use the term "prioritization" because "paid prioritization" is the standard way to refer to this type of quality differentiation, although in broad terms zero-rating is also a form of prioritization. Moreover, we use "prioritization" instead of "paid prioritization" because we allow the ISP to prioritize a CP even in the absence of transfers between the ISP and the CP.

[4]We use the term "zero-rating" instead of "sponsored data" because the latter implies a transfer from the CP to the ISP and our model allows for zero-rating even in the absence of such transfers.

to be in a monopolistic position.

The ISP faces a capacity constraint and thus may not be able to deliver the best available quality of content to all users. The comparison of the three regulatory regimes is meaningful only under the risk of congestion when the discriminatory programs are likely to be offered. Therefore our analysis mainly applies to the mobile internet market where limited transmission capacity is still an important constraint for ISPs. In our model, the price charged by the ISP to users, the market shares of CP and the download qualities are all endogenous. In this context, both P and ZR constitute a way to differentiate content further for the users.

Solving the model, we show that zero-rating always leads to higher ISP prices than prioritization. However, these higher prices do not necessarily lead to lower consumer surplus for the following reason. Both commercial practices favor one CP over the other, thus increasing the market share of the privileged CP at the expense of its competitor. Thus they both lead to a distortion in CPs' market shares compared to the neutral situation. As under net neutrality the total transportation cost borne by consumers is minimal, the larger the distortion between CPs market share, the more transportation costs they must pay. We show that under some conditions prioritization of content results in a larger distortion than zero-rating, and the resulting increased transportation costs can lead to a lower consumer surplus under prioritization, despite the lower price.

Creating distortions on the content market is a tool for the ISP to extract payments from content providers. The larger the distortions, the more valuable is the privileged content and thus the larger the payment to the ISP. We show that when the value of traffic for content providers (the advertising revenue per viewer) is limited, the ISP's optimal policy is zero-rating. When the value of traffic is high, the optimal policy is paid prioritization because the ISP can extract a lot from the privileged content provider. In this case, the interests of the consumers and the ISP may be aligned, both preferring prioritization of traffic to zero-rating or a neutral internet.

1.1 Related literature

Our research project is closely related to the rich body of literature in theoretical industrial organization about net neutrality. With a few exceptions (see e.g., Broos and Gautier (2017)) these articles have focused on the effect of throttling and paid prioritization. Some seminal articles in the topic are Hermalin and Katz (2007); Choi and Kim (2010); and Economides and Hermalin (2012). For a survey of paid prioritization, see Greenstein et al. (2016) or Krämer et al. (2013). Some recent research about the topic includes Bourreau et al. (2015); Choi et al. (2015); Peitz and Schuett (2016); and Reggiani and Valletti (2016).

The only formal economic studies modeling zero-rating to date are Jullien and Sand-Zantman (2017); and Somogyi (2016). The analysis of zero-rating has mostly been relegated to the realms of legal science and descriptive studies. For two recent summaries providing an overview of zero-rating programs and the current state of regulation, see Marsden (2016) and Yoo (2016). The FCC's and BEREC's public consultations gave rise to numerous advocacy papers discussing either the merits or the drawbacks of zero-rating programs, for instance Eisenach (2015), the ITIF report by Brake (2016) and the WWW Foundation's report by Drossos (2015). For a rather impartial overview of the main arguments, see CDT's report by Stallman and Adams (2016). Arguably the most comprehensive set of arguments against zero-rating is presented by van Schewick (2016), while the most comprehensive (informal) economic presentation of its merits is by Rogerson (2016).

The fact that congestion is still a salient feature of the internet today has recently been documented by the thorough empirical studies of Nevo, Turner, and Williams (2016) and Malone, Nevo, and Williams (2017). In particular, by analyzing fixed residential broadband data they estimate a large willingness-to-pay to avoid congestion. Several factors, including hard technological constraints make mobile internet even more susceptible to congestion, including the scarcity of spectrum and limits on cell size reduction (see e.g. Clarke (2014)). Indeed, Heikkinen and Berger (2012) find that in most countries packet loss, a standard measure of congestion, is higher on the mobile internet.

2 Model

There are three categories of players in the model: competing content providers (CP), a monopolistic internet service provider (ISP) and consumers.

2.1 Content providers

There are two contents providers and we use an Hotelling line of size 1 to model differentiation between contents. CPs are asymmetric, CP_1 is located at point $a \geq 0$ on the unit interval and $CP2$ is located at point 1. CP_1, hereafter the strong content, is in a privileged position with more consumers closer to it than its rival CP_2, hereafter the weak content. In the sequel, we will assume that $a < \frac{1}{7}$.

Consumers who are connected to the internet chooses to consume content 1 or content 2. Contents are offered for free by the CP and the content provision service is financed by advertising revenues. We suppose that the CPs collect an (exogenous) advertising revenue ρ per active viewer/subscriber. Costs for the CP are normalized to zero. If we denote the total number of viewers of content i by n_i, the profits of CP_i, $i = 1, 2$ are: $\pi_{CP_i} = \rho n_i$.

2.2 Internet service provider

There is a single ISP. Consumers need to be connected to the ISP to access the contents. Each connected consumer will consume one unit of content either at CP_1 or at CP_2. The total demand of content is thus $n_1 + n_2$.

The ISP has a bandwidth capacity κ. Limited capacity may lead to congestion if the bandwidth is insufficient to carry on all the traffic. The consumption of time-sensitive content will be altered in case of congestion and consumers will experience jitters, delays, interruptions or a degradation of the content quality (throttling). We will represent the surfing experience quality by a parameter q, that can be interpreted as the probability of on-time delivery (Peitz and Schuett, 2016). This probability is equal to the ratio of the bandwidth and the traffic: $q = Min[1, \frac{\kappa}{n_1+n_2}]$. If $n_1 + n_2 < \kappa$, there is no congestion on the internet and all contents can be delivered on time: $q = 1$.

If $n_1 + n_2 > \kappa$, the internet is congested and the ISP can no longer deliver the quality $q = 1$ to all users. If there is no discrimination between contents, the probability of on-time delivery is the same for all users and contents and equal to $q = \frac{\kappa}{n_1+n_2} < 1$.

In the prioritization regime, the ISP will give priority to content i over content j and the probability of on-time delivery will be higher for content i than for content j: $q_i \geq q_j$. If $n_i < \kappa$, part of the bandwidth will be used for the prioritized content that will be delivered on time: $q_i = 1$ while the remaining part will be used for the non-priority content that will experience delays: $q_j = \frac{\kappa - n_i}{n_j} < 1$. If $n_i > \kappa$, it is not possible to deliver the the priority content on-time and the probabilities q_i and q_j are $q_i = \frac{k}{n_i} > 0$ and $q_j = 0$. In both cases, the quality differential decreases with the bandwidth capacity κ:[5] $\frac{\partial(q_i - q_j)}{\partial \kappa} \leq 0$.

Consumers will consume at most one unit of content, either at CP_1 or CP_2 and for that they must pay a connection fee to access the internet. We will suppose that the ISP applies a two-part tariff with a fixed fee p and a price per unit of content, that we call d. This formulation is convenient to model zero-rating. Zero-rating practices consist in applying different prices for different content consumption by excluding the eligible CPs data from users monthly data cap. We will model zero-rating by assuming that the ISP charges a basis price p for accessing the internet and a unit fee d_i for downloading the content CP_i, with $d_i = 0$ for the zero rated content. The revenue for the ISP from the consumer side is equal to $\sum_{i=1,2}(p + d_i)n_i$.

2.3 Consumers

The utility of a consumer when he chooses content i depends on (1) the price for accessing the content, (2) his preference for content i over content j, this horizontal differentiation is captured by his position on the Hotelling line, and (3) the download quality of the content, q_i. When $q_i \neq q_j$, we have an additional level of vertical differentiation between the different types of contents.

There is a mass one of consumers located on the Hotelling line. If we denote by x the location of the consumer on the line and by τ the unit transportation cost, the

[5] This important property is also present in the M/M/1 queuing system used by Choi and Kim (2010) to model congestion on the internet.

utility of consumer x is defined as:

$$U(x) = \begin{cases} v + q_1 - \tau x - (p + d_1) & \text{if he chooses } CP_1 \\ v + q_2 - \tau(1 - x) - (p + d_2) & \text{if he chooses } CP_2 \end{cases}$$

In the basic framework with a mass one of consumers we will suppose that (1) the market is fully covered at equilibrium: $n_1 + n_2 = 1$, a sufficient condition for that is a sufficiently large value for v, (2) there is not enough download capacity : $\kappa < 1$, and (3) consumers have a reservation utility of zero.

2.4 Regulatory regime

The ISP can make two types of discrimination between contents: quality discrimination and financial discrimination. Quality discrimination consists in differentiating contents by having a different probability of on-time delivery ($q_1 \neq q_2$). Financial discrimination consists in charging a higher price for the consumption of a particular content ($d_1 \neq d_2$).

We will consider two different regulatory regimes: the net neutrality and the laissez-faire. Under Net Neutrality (NN), both types of discrimination are prohibited (by law). In a neutral internet, we have $q_1 = q_2$ and $d_1 = d_2$ and for simplicity we set $d_1 = d_2 = 0$. In the laissez-faire regime, the ISP can discriminate in price or in quality between different types of content.[6] There are two different regimes in a laissez-faire economy: Prioritization (P) and Zero-rating (ZR). Under prioritization, the ISP gives priority to CP_i over CP_j and creates a vertical differentiation between the two contents $q_i \geq q_j$. In this regime, the price paid by the consumers is the same: $d_1 = d_2 = 0$. Prioritization can be given for free or in exchange for a compensation from the prioritized content provider. When the content provider compensates the ISP for priority access, it is referred to as paid prioritization. Under zero-rating, there is no vertical differentiation between contents: $q_1 = q_2 = q$ but the ISP financially discriminates between the zero-rated content for which $d_i = 0$ and the non zero-rated content for which $d_j > 0$. Zero-rating can be offered for free or against a compensation.[7]

[6] For the moment, we do not consider the possibility to discriminate at the same time in price and in quality.

[7] In the US, most of the ISP zero-rate some video-content. The Binge On program of T-Mobile is a

2.5 Timing of the events and equilibrium concept

The timing of the events is the following:

1. In the laissez-faire regime, the ISP decides to implement P, ZR or nothing (which is equivalent to NN).
2. Under P or ZR, the content providers compete for priority.
3. The ISP sets the connection price(s) p (and d_i under ZR).
4. Consumers simultaneously decide which CP to buy from, correctly anticipating the decision of other consumers.

We will refer to stages 3 and 4 of the full game as the *pricing subgame*. There are five different pricing subgames corresponding to the following regimes: net neutrality, prioritization giving priority to either the strong or the weak firm, or zero-rating the content of either the strong or the weak firm. In this model, the eventual compensation paid by the CP to the ISP for being privileged is a pure transfer between the ISP and the CP and does not influence the pricing subgame.

Under NN and ZR, the quality of content the consumers face is independent of the action of other consumers. However, under P, the utility of accessing prioritized content decreases in the mass of other consumers subscribing to the priority content. Indeed, in the extreme case of everyone subscribing to the prioritized content, the quality they enjoy is the same as under the other regimes, i.e. all advantage of buying prioritized content disappears. Therefore, when making their decision in stage 4 of the game, consumers are part of a game involving all other consumers. We apply the concept of rational expectations equilibrium to solve this subgame, i.e. we assume that consumers can correctly anticipate the decision of other consumers. From a technical viewpoint, this means that we will find the location of indifferent consumers as fixed points of the demand functions. Note that this assumption is common in the literature (see e.g. Choi and Kim, 2010). As usual, we will solve the full game by backwards induction using the solution concept of subgame-perfect Nash equilibrium.

free zero-rating program and all the content providers that meet the technical requirement can apply freely. On the contrary, to be admitted in the zero-rating plan of AT&T (called Sponsored Data), content providers have to compensate the ISP.

3 Equilibria of the pricing subgame

In this section we will derive equilibria of the pricing subgame that consists of the ISP's pricing decision followed by the consumers' choice of content. We will find equilibria under the five possible regimes.

3.1 Net neutrality

Under net neutrality there is a unique price p to access both CPs' content and their quality q are identical. The indifferent consumer between CP_1 and CP_2 is characterized by:

$$v + q - p - \tau(x - a) = v + q - p - \tau(1 - x) \quad \Leftrightarrow \quad x = \frac{a + 1}{2}.$$

Using $q = \kappa$, the ISP maximizes its profit by extracting all the surplus from the indifferent consumer and chooses a price

$$p^{NN} = v + \kappa - \frac{\tau}{2} + \frac{a\tau}{2}.$$

If $a \leq 1/3$, the consumer located at 0 is willing to buy at that price and the market is fully covered. The profits of the ISP and the CPs are equal to $\pi_{ISP}^{NN} = p^{NN}$, $\pi_{CP_1}^{NN} = \rho\frac{a+1}{2}$ and $\pi_{CP_2}^{NN} = \rho\frac{1-a}{2}$.

3.2 Zero rating

Under ZR, the two contents have the same download quality $q = \kappa$ but they are financially differentiated. Accessing the zero-rated content costs p while accessing the other costs $p + d$. The ISP has the choice between zero-rating the strong or the weak content but the later option is always strictly preferred.

When CP_2 is zero rated, the indifferent consumer is located at:

$$v + \kappa - p - d - \tau(x - a) = v + \kappa - p - \tau(1 - x) \quad \Leftrightarrow \quad x = \frac{1 + a}{2} - \frac{d}{2\tau}.$$

The ISP's maximization problem is the following:

$$\max_{p,d} \quad p + xd = p + \frac{1+a}{2}d - \frac{d^2}{2\tau} \qquad \text{s.t.} \qquad p \le v + \kappa - \tau(1-x)$$

The optimal solution is $p^{ZR} = v + \kappa - \frac{\tau}{2} + \frac{a\tau}{4}$ and $d = \frac{a\tau}{2}$. At this price, the market is covered provided that $a \le 2/7$. Notice that the price charged for zero-rated content is lower than the uniform price under net neutrality, whereas the non-zero-rated content is more expensive: $p^{ZR} < p^{NN} < p^{ZR} + d$. These prices lead to the indifferent consumer being located at $x = \frac{1}{2} + \frac{a}{4}$. The average price $\tilde{p}^{ZR}$ and the ISP's profitare:

$$\tilde{p}^{ZR} = \pi_{ISP}^{ZR} = v + \kappa - \frac{\tau}{2} + \frac{a\tau}{2} + \frac{a^2\tau}{8} = \pi^{NN} + \frac{a^2\tau}{8}.$$

Zero-rating the weak firm is profitable for the ISP whenever $a > 0$ and it is easy to check that zero-rating the strong firm gives lower profits. The profits of the CPs are: $\pi_{CP_1}^{ZR} = \rho\frac{2+a}{4}$ and $\pi_{CP_2}^{ZR} = \rho\frac{2-a}{4}$.

3.3 Prioritization

With prioritization, the two content providers offer different qualities. If content i has the priority, the qualities are $(q_i, q_j) = (1, \frac{\kappa - n_i}{n_j})$ if $n_i \le \kappa$ and $(q_i, q_j) = (\kappa, 0)$ otherwise. Qualities depend on market shares and according to the rational expectations equilibrium concept we assume that in equilibrium consumers correctly anticipate the content qualities.

There are three possible equilibrium configurations of the consumer subgame (the last stage of the full game). When content i has priority, we call

1. an *interior equilibrium* when $n_i, n_j > 0$ and $(q_i, q_j) = (1, \frac{\kappa - n_i}{n_j})$,
2. a *semi-corner equilibrium* when $n_i, n_j > 0$ and $(q_i, q_j) = (\kappa, 0)$, and
3. a *corner equilibrium* when $n_j = 0$.

In a corner equilibrium, the market tips in the sense that all consumers choose the prioritized firm. Such an equilibrium arises typically when the transportation cost τ is very low. In a semi-corner equilibrium, some consumers choose the non-prioritized firm even though it offers the lowest quality (recall that we assume a large valuation v for internet connection that is irrespective of the quality of the CPs). Such an equilibrium arises typically when congestion is very severe, i.e. the capacity κ is small. As usual in the literature, we will focus our attention on interior equilibria. Firstly, we derive conditions of existence of such equilibria both for the case when the weak and the strong CP is prioritized. Secondly, we show that the interior equilibria when the weak firm is prioritized is the unique equilibrium of the subgames involving prioritization whenever such an equilibrium exists.

Priority to the weaker CP

If the ISP gives priority to the weaker firm's content, in an interior equilibrium the indifferent consumer's location x is given by

$$v + q_1 - p - \tau(x - a) = v + 1 - p - \tau(1 - x),$$

where the quality of the weaker CP's content is $q_1 = \frac{\kappa-(1-x)}{x} = 1 - \frac{1-\kappa}{x}$, given that x is indeed an interior solution, i.e. $1 - x < \kappa$. This leads to two potential interior solutions for x:

$$x_1 = \frac{1+a}{4} - \sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}} \quad \text{and} \quad x_2 = \frac{1+a}{4} + \sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}},$$

which in turn lead to two different prices:

$$p_1 = v+1-\tau\left(\frac{3-a}{4} + \sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}}\right) < p_2 = v+1-\tau\left(\frac{3-a}{4} - \sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}}\right).$$

Given that $x_1 < x_2$, it is straightforward that the latter leads to a higher price. Moreover, whenever $1 - x_1 < \kappa$, the condition for the first solution being interior is satisfied, $1 - x_2 < \kappa$ also holds, i.e. the second solution is also interior. Therefore, whenever the first solution is potentially an interior equilibrium, the second solution is also potentially an interior equilibrium and it leads to a higher price for the ISP.

Hence the first solution is always dominated by the second and will never constitute an equilibrium of the pricing subgame. The following Lemma establishes the conditions for the existence of an interior equilibrium when the weak firm is prioritized.

Lemma 1 *An interior solution for the pricing subgame exists under the regime when the weaker CP has priority whenever* $\kappa \geq \frac{1+\tau-a\tau}{2\tau}$. *It consists of the ISP choosing a price equal to* $p_W^{INT} = v + 1 - \tau\left(\frac{3-a}{4} - \sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}}\right)$, *with the indifferent consumer located at* x_2.

Thus an interior equilibrium exists if the capacity is sufficiently large and the two CPs are sufficiently differentiated horizontally. Intuitively, a large capacity is needed for the firm to be able to (profitably) serve every consumer and thus avoid a semi-corner solution. Moreover, products must be sufficiently different from each other to avoid market tipping, a corner solution in which the prioritized firm serves all consumers.

Note that the condition for the consumer sitting at 0 buying from CP_1 is $x_2 \geq 2a$ which is satisfied for $a \leq 1/7$.

Priority to the stronger CP

If the ISP gives priority to the stronger firm's content, in any interior solutions the indifferent consumer's location x is given by

$$v + 1 - p - \tau(x-a) = v + q_2 - p - \tau(1-x) \quad \Leftrightarrow \quad x = \frac{1+a}{2} + \frac{1-q_2}{2\tau},$$

where the quality of the weaker CP's content is $q_2 = \frac{\kappa - x}{1-x}$. As in the previous case, there are two potential interior solutions for x:

$$x_1' = \frac{3+a}{4} - \sqrt{\frac{(1-a)^2}{16} - \frac{1-\kappa}{2\tau}} \quad \text{and} \quad x_2' = \frac{3+a}{4} + \sqrt{\frac{(1-a)^2}{16} - \frac{1-\kappa}{2\tau}},$$

which in turn lead to two different prices for the ISP:

$$p_1' = v+1-\tau\left(\frac{3(1-a)}{4} - \sqrt{\frac{(1-a)^2}{16} - \frac{1-\kappa}{2\tau}}\right) > p_2' = v+1-\tau\left(\frac{3(1-a)}{4} + \sqrt{\frac{(1-a)^2}{16} - \frac{1-\kappa}{2\tau}}\right).$$

Analogously to the previous case, it can be shown that one of the solutions leads to a strictly higher price while being always an interior solution whenever the other solution is interior. Therefore, the solution x_2' is dominated, and the condition for the existence of an interior solution is $x_1' < \kappa$ which translates to $\kappa \geq \frac{1+\tau+a\tau}{2\tau}$. In the following, we will use the following notations to stress that the values belong to the strong CP: $x_S^P = x_1'$ and $p_S^P = p_1'$.

Comparison of equilibria under prioritization

Next, we establish a clear relationship between all the equilibria of the subgames in which the ISP opts for P. In particular, we show in the Appendix that whenever an interior equilibrium in the subgame prioritizing the strong CP exists, an interior equilibrium in the subgame prioritizing the weak CP also exists. Moreover, this latter always results in a higher price for the ISP, therefore the former equilibrium can never be a subgame-perfect equilibrium of the full game. The next Proposition establishes an even stronger claim: The interior equilibrium when the weak CP is prioritized dominates all other potential equilibria when P is chosen in the first stage, i.e. it dominates all the potential corner and semi-corner equilibria as well.

Proposition 1 *The interior solution of the pricing subgame with the weaker CP having priority leads to a higher price than all other potential equilibria of the subgames involving prioritization whenever such an equilibrium exists, i.e. for* $\kappa \geq \frac{1+\tau-a\tau}{2\tau}$.

The proof, relegated to the Appendix, compares the price p_W^{INT} to all other equilibria involving prioritization, including semi-corner and corner equilibria, both when the weak and the strong firm is prioritized, and shows that it is the highest (whenever its existence condition is satisfied). Consequently, the equilibrium price under P and the

ISP's profit are:

$$p^P = v + 1 - \tau \left(\frac{3-a}{4} - \sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}} \right) = \pi^P_{ISP}.$$

At this equilibrium, the indifferent consumer is located at $x_2 = x^P_W = \frac{1+a}{4} + \sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}}$ and the CPs' profits are: $\pi^P_{CP_1} = \rho(1 - x_2)$ and $\pi^P_{CP_2} = \rho x_2$.

In the following, we will assume $\kappa \geq \kappa^* = \frac{1+\tau-a\tau}{2\tau}$. According to Proposition 1, given this assumption it is sufficient to compare p^P to the equilibrium profits arising from ZR and NN to find the equilibrium of the full game. Remark that $\kappa^* < 1$ implies that $\tau(1 + a) > 1$.

4 Comparisons

In this section, we compare the equilibria in the three different price subgames.

4.1 Prices

We start our comparisons by focusing on the price charged by the ISP (the weighted average price in the zero-rating case).[8]

The price p^{NN} and the weighted average price $\tilde{p}^{ZR}$ are both linearly increasing in the capacity level κ. In both cases, the quality offered is uniform. Therefore a capacity increase has the same impact on all consumers and the monopolist manages to capture completely this increased surplus by charging a higher price. A higher capacity has no impact on the consumers' surplus but only on the ISP's profit. Comparing the two prices, it is immediate that $\tilde{p}^{ZR} > p^{NN}$, $\forall \kappa$. Financial discrimination between contents is always profitable for the ISP, even if there is no congestion ($\kappa = 1$). This result does not come as a surprise.

The price p^P is not linear but concave in κ. With prioritization, quality is not uniform and a capacity increase benefits only to those who are consuming the non-

[8] If there is no side payment between the privileged content and the ISP, the comparison of prices is equivalent to the comparisons of the ISP's profits.

prioritized content. And, this benefit is larger when there are fewer consumer at the non-prioritized content, in other words, the quality q_1 is increasing and concave in k. An increase in q_1 makes the non-prioritized content more attractive relative to the prioritized one. The indifferent consumer shifts to the right and the ISP collects the surplus of the indifferent consumer. Concavity of price means that the additional surplus decreases with κ.

Comparing the prices $\tilde{p}^{ZR}$ and p^P, we show that: $\tilde{p}^{ZR} \geq p^P$ for all $\kappa \in [\kappa^*, 1]$ with a strict inequality for $\kappa \neq \kappa_1 = 1 - \frac{a\tau(2+a)}{8}$. The ISP charges always a higher price under ZR than under P. Prices are identical for a particular value $\kappa = \kappa_1$. For this capacity constraint, the quality differential under P ($= 1 - q_1$) is equal to the price differential under ZR ($= d$). Therefore, for this value price and quality differentiations are equivalent for the ISP, the consumers and the CPs.

Comparing the prices under NN and P, we found that p^P is larger than p^{NN} if $\kappa \geq \kappa_2 = 1 - \frac{a\tau}{2}$, with $\kappa_2 < \kappa_1$. We summarize our findings in the following lemma:

Lemma 2 *(1) If $(\tau - 1)(1 - a\tau) > 1$, then $p^{ZR} \geq p^P \geq p^{NN}$ for all $\kappa \in [\kappa_2, 1]$ and $p^{ZR} \geq p^{NN} \geq p^P$ for all $\kappa \in [\kappa^*, \kappa_2]$.*

(2) If $(\tau - 1)(1 - a\tau) < 1$, then $p^{ZR} \geq p^P \geq p^{NN}$ for all $\kappa \in [\kappa^*, 1]$.

The results of Lemma 2 are represented on figure 1.

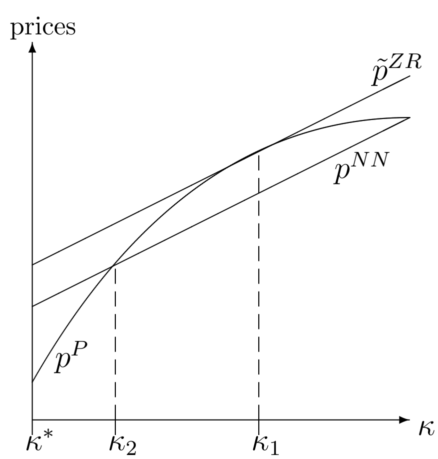

Figure 1: Prices under NN, ZR and P for $(\tau - 1)(1 - a\tau) > 1$.

4.2 Consumer surplus and welfare

We now turn to the comparison of the consumer's surplus. To understand our comparison, it is important to remind two important features of the model. First, the average quality is the same in all the configurations we considered (and equal to κ), and in particular, there is no improvement of the average quality under prioritization. Second, transportation costs are minimized if there is no discrimination between contents i.e. they are the lowest under net neutrality.[9]

These observations lead to the immediate conclusion that the consumers' surplus is higher under NN than under ZR. Furthermore, the prioritization regime cannot be optimal when the ISP charges a higher price compare to net neutrality. In other words, the consumer surplus is the highest with NN for $\kappa \geq \kappa_2$.

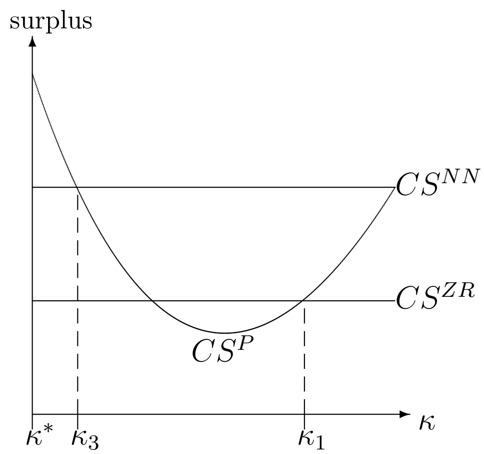

Figure 2: Consumer surplus under NN, ZR and P.

The consumer surplus (CS) under P is U-shaped with $CS^P = CS^{NN}$ for $\kappa = 1$. When κ decreases, there are two effects at play. First, the price paid by consumers decreases (see Figure 1 above). Second, the quality differential $(1 - q_1)$ increases. This second effect creates distortions on the content market and the aggregate transportation cost increases. The consumer surplus under P, ZR and NN is represented on Figure 2. For $\kappa = \kappa_1$, the price differential is equal to the quality differential and the surplus is the same under P and ZR. For $\kappa < \kappa_1$, the consumer surplus is higher under ZR than under

[9]Together it implies that welfare maximization is equivalent to the minimization of transportation costs.

P despite a higher (average) price paid by the consumers to the ISP. The reason is that there are more distortions on the content market –hence higher transportation costs– as the quality differential is larger than the price differential d. For lower values of κ, the price effect is more important than the quality differential effect and the consumer surplus under P is decreasing in κ. Eventually, the surplus under P can be larger than the surplus under NN. For that, the price differential $p^{NN} - p^P > 0$ for $\kappa < \kappa_2$ should more than compensate the distortions in the content market. A necessary condition for a higher surplus under P is $\kappa_2 \geq \kappa^*$, corresponding to the conditions (1) of Lemma 2. Comparing the surplus under NN and P, we found that CS^P is larger than CS^{NN} if $\kappa \leq \kappa_3 = 1 - a\tau + \alpha\tau^2 < \kappa_2$. We summarize our findings in the following lemma:

Lemma 3 *(1) If $\kappa_3 \leq \kappa^*$, the the consumer surplus is the highest under net neutrality for all $\kappa \in [\kappa^*, 1]$.*

(2) If $\kappa_3 \geq \kappa^*$, $CS^P \geq CS^{NN}$ for $\kappa \in [\kappa^*, \kappa^3]$ and $CS^P \leq CS^{NN}$ for $\kappa \in [\kappa^3, 1]$.

In the Appendix, we have the proof of the Lemma and the detailed expressions for the surplus.

From the two above lemmas, it is clear that consumers and the ISP have opposed interest. First, the ISP prefers ZR to NN while consumers prefer NN to ZR. Consumers and the ISP are totally opposed regarding financial discrimination (see Figure 3). Second, if we suppose that $\kappa^* \leq \kappa_3$, then we have $p^P \leq p^{NN}$ and $CS^P \geq CS^{NN}$ for $\kappa \in [\kappa^*, \kappa_3]$ (opposed interests), $p^P \leq p^{NN}$ and $CS^P \leq CS^{NN}$ for $\kappa \in [\kappa_3, \kappa_2]$ (congruent interest) and $p^P \geq p^{NN}$ and $CS^P \leq CS^{NN}$ for $\kappa \in [\kappa_2, 1]$ (opposed interest).

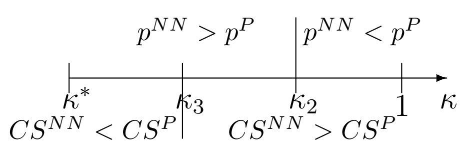

Figure 3: Prices and consumer surplus under NN and P.

The total welfare is defined as the sum of $CS + \pi^{ISP} + \sum_{i=1,2} \pi^{CP_i}$. Welfare maximization is in our model equivalent to the minimization of transportation costs. Transportation costs are minimized when $x = \frac{1+a}{2}$ and this solution is implemented under net neutrality. Under P and ZR, the strong firm has a market share lower than $\frac{1+a}{2}$; the solution leading to the highest welfare corresponds to the case where the strong firm's market share is the largest. The solution that imposes the smallest distortion in the content market is the one leading to the highest welfare. Thus for $\kappa > \kappa_1$, there are less distortions with P and the welfare is higher compared to ZR while for $\kappa < \kappa_1$, it is the opposite.

5 Potential transfers from the CPs to the ISP

In the previous section we have assumed that the ISP cannot ask for compensation neither for prioritization of content nor for zero-rating. In this section we relax this assumption and investigate how the ISP's ability to extract money from the CPs alters the outcomes of the game.

Importantly, we assume that in the beginning of the game the ISP credibly commits itself to offering only one type of preferential treatment: either paid prioritization or zero-rating, and this becomes common knowledge among all players. Therefore, after committing to paid prioritization, it cannot threaten a CP to approach its competitor with a zero-rating offer and vice versa. In other words, we restrict the ISP's possible threats in the bargaining stage, which greatly simplifies the analysis.

5.1 Zero-rating offers

In this subsection we investigate the outcome of a subgame in which the ISP is restricted to zero-rating offers.

In the previous analysis we established that in the absence of transfers from CPs to the ISP, the optimal zero-rating policy of the ISP is zero-rating the weak CP. It also clearly follows that the ISP achieves a higher profit by not implementing any zero-rating than by zero-rating the strong CP. Thus it prefers the net neutrality situation over zero-rating the strong firm and this is common knowledge.

Therefore the credible threat point of the ISP in case its offer is refused by the weak CP is the NN situation. The weaker CP loses the market share of $(1-x^{ZR})-(1-x^{NN})$, thus its maximal willingness to pay for the preferential treatment is $\rho(x^{NN}-x^{ZR})$.

The strong CP's maximal bid is the same amount, as it knows that it is never in the ISP's interest to zero-rate its content. Its loss of revenue from its competitor being zero-rated instead of the NN situation is exactly $\rho(x^{NN}-x^{ZR})$.

There are two cases depending on the size of the CPs' advertising revenue, ρ. First, if it is sufficiently large that the maximal bid compensates the ISP for the loss of revenue, i.e., if

$$\rho(x^{NN}-x^{ZR})+p^{NN}>\tilde{p}^{ZR} \quad \Leftrightarrow \quad \rho>\frac{\tilde{p}^{ZR}-p^{NN}}{x^{NN}-x^{ZR}}\equiv \rho^{ZR} \tag{1}$$

then the weaker firm must bid a positive amount to get the zero-rating agreement. In particular, in equilibrium the weak CP's bid equals

$$\rho(x^{NN}-x^{ZR})+p^{NN}-\tilde{p}^{ZR}>0$$

which is positive given $\rho>\rho^{ZR}$ and smaller as its maximal willingness to bid as $\tilde{p}^{ZR}>p^{NN}$.

Second, if advertising revenues are relatively small, $\rho\leq\rho^{ZR}$, then (1) is not satisfied, thus the stronger CP's maximal bid is insufficient to compensate for the ISP's loss of profits, therefore the weak firm gets zero-rated for free, thus its optimal bid is 0.

Generally, the weak CP bids $\max\{0;\ \rho(x^{NN}-x^{ZR})+p^{NN}-\tilde{p}^{ZR}\}$ and wins the zero-rating contract. The ISP's overall profit is thus given by $\max\{\tilde{p}^{ZR};\ \rho(x^{NN}-x^{ZR})+p^{NN}\}$.

5.2 Paid prioritization offers

In this subsection we investigate the outcome of a subgame after the ISP chooses paid prioritization offers in the previous stage.

Although from the previous section we know that under $\kappa \geq \kappa^*$ prioritizing the weak CP is the best prioritizing option for the ISP, the payment the ISP can extract from the CPs depend on the exact value of its profit when prioritizing the strong CP. The next Lemma identifies a necessary and sufficient condition for the interior solution to be optimal for the ISP in case it offers priority to the strong CP, analogously to Lemma 1 and Proposition 1.

Lemma 4 *The interior solution of the pricing subgame with the* stronger *CP having priority leads to a higher price than all other potential equilibria of the subgames involving prioritizing the strong CP whenever such an equilibrium exists, i.e. for* $K \geq \frac{1+\tau+a\tau}{2\tau}$*. The ISP's profit is then given by* p_S^P*.*

Notice that the condition of Lemma 4 is stronger than the condition of Proposition 1, but they are qualitatively similar: they both require a relatively high level of capacity and horizontal differentiation. Three cases have to be distinguished according to the relative sizes of p^{NN}, p^P, and p_S^P.

Case 1: $\kappa > \kappa_2$ **and** $p_S^P > p^{NN}$.

In this case p^{NN} is the smallest of the three values, therefore the ISP is better-off offering prioritizing the strong firm's content than having the neutral situation. Following the logic laid out above in the zero-rating case, we first determine the strong firm's maximal bid. Its maximal bid equals $\rho(x_W^P - x_S^P)$. The weak CP must then bid

$$\max\{0;\ \rho(x_W^P - x_S^P) + p_S^P - p^P\}$$

to win the contract, and the ISP's profit is given by $\max\{p^P;\ \rho(x_W^P - x_S^P) + p_S^P\}$.

Case 2: $\kappa > \kappa_2$ **and** $p_S^P \leq p^{NN}$.

The only difference with respect to the previous case is that the ISP is better-off not implementing any prioritization than giving priority to the strong firm. The strong CP's maximal bid is thus reduced to $\rho(x^{NN} - x_W^P)$. Therefore, following the previous steps, in equilibrium the weak CP wins the paid prioritization contract by bidding $\max\{0;\ \rho(x^{NN} - x_W^P) + p^{NN} - p^P\}$ and the ISP's profit is $\max\{p^P;\ \rho(x^{NN} - x_W^P) + p^{NN}\}$.

Case 3: $\kappa \leq \kappa_2$.

If the level of capacity is below κ_2 then $p_S^P < p^P < p^{NN}$, i.e. in the absence of sufficiently large payments from the CPs the ISP prefers not to prioritize any of them. Let b^S denote the maximal bid of the strong firm, $\rho(x^{NN} - x_S^P)$. Similarly, let b^W be the maximal bid of the weak firm, $\rho(x_W^P - x^{NN})$. If $b^W < p^{NN} - p^P$ and $b^S < p^{NN} - p_S^P$ are jointly satisfied then none of the CPs find it profitable to compensate the ISP's loss from prioritization, therefore net neutrality arises as an equilibrium outcome.

If at least one of these inequalities is not satisfied than net neutrality will surely be overturned by a large enough bid. Therefore the maximal bid of the strong firm is $\rho(x_W^P - x_S^P)$. Following the usual arguments, in equilibrium the weak firm secures the paid priority contract by bidding $\max\{0;\ \rho(x_W^P - x_S^P) + p_S^P - p^P\}$ and the ISP's profit is $\max\{p^P;\ \rho(x_W^P - x_S^P) + p_S^P\}$.

5.3 ISP's choice between paid prioritization and zero-rating

Next we investigate the ISP's decision in a regulatory environment where the ISP is free to choose either prioritization or zero-rating.

The level of CPs' advertising revenues is a key factor in the ISP's choice. From the previous results it clearly follows that for low advertising revenues the ISP prefers zero-rating as $\tilde{p}^{ZR} > p^P$. Intuitively, with low advertising revenues, the CPs would not find it profitable to compensate the ISP for switching to the other regime.

Conversely, with high advertising revenues the large payments from CPs to the ISP can dwarf the loss of ISP profit when it changes to paid priority programs. These programs create a larger divide in market shares than zero-rating programs, making them more attractive for the ISP when advertising revenues are high. The next Proposition formalizes these findings.

Proposition 2 *For any $\kappa > \kappa^*$ there exists a threshold level of advertising revenues, $\overline{\rho}(\kappa) > 0$, such that the ISP implements a zero-rating with the weak CP for $\rho \leq \overline{\rho}(\kappa)$, otherwise it provides paid priority to the weak CP.*

We derive the exact value of $\overline{\rho}(\kappa)$ as a function of the parameters in the Appendix.

Appendix

Proof of Lemma 1 There are two conditions for the existence of an interior solution of the pricing subgame when the weaker CP has priority. The first is that the quadratic equation defining the location of the indifferent consumer have real root(s), which is guaranteed if and only if

$$\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau} \geq 0 \quad \Leftrightarrow \quad \kappa \geq 1 - \frac{\tau(1+a)^2}{8}.$$

The second condition is that distance between the indifferent consumer and the weaker CP is less than κ, which rewrites as

$$\frac{1+a}{4} + \sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}} \geq 1 - \kappa \quad \Leftrightarrow \quad \kappa \geq \frac{1+\tau-a\tau}{2\tau}.$$

The second condition implies the first one as $\frac{1+\tau-a\tau}{2\tau} > 1 - \frac{\tau(1+a)^2}{8}$, thus the second condition is necessary and sufficient for the existence of an interior solution of the pricing subgame when the weaker CP has priority. ■

Proof of Proposition 1 In order to prove the Proposition, we will compare p_W^{INT} to all other potential equilibrium prices under the condition $\kappa \geq \frac{1+\tau-a\tau}{2\tau}$. There are six other potential equilibria: the interior solution when the strong CP is prioritized derived in the main text, the two corner equilibria (one where the strong, one where the weak CP has priority), and three semi-corner equilibria (the one where the strong CP has priority, and two where the weak CP has priority, depending on whether $x \lesseqgtr a$).

Interior equilibrium when the strong CP is prioritized: the Proposition holds if and only if the price derived in the main text, p_S^{INT} is below p_W^{INT}, i.e.

$$v+1-\tau\left(\frac{3(1-a)}{4}-\sqrt{\frac{(1-a)^2}{16}-\frac{1-\kappa}{2\tau}}\right)<v+1-\tau\left(\frac{3-a}{4}-\sqrt{\frac{(1+a)^2}{16}-\frac{1-\kappa}{2\tau}}\right).$$

Clearly, this is equivalent to

$$\frac{3(1-a)}{4}-\sqrt{\frac{(1-a)^2}{16}-\frac{1-\kappa}{2\tau}}>\frac{3-a}{4}-\sqrt{\frac{(1+a)^2}{16}-\frac{1-\kappa}{2\tau}}.$$

which rewrites as

$$a/2>\sqrt{\frac{(1+a)^2}{16}-\frac{1-\kappa}{2\tau}}-\sqrt{\frac{(1-a)^2}{16}-\frac{1-\kappa}{2\tau}}.$$

After squaring both sides of the inequality (both sides being positive) and rearranging, we get

$$\frac{(1-a)(1+a)}{16}-\frac{1-\kappa}{2\tau}>\sqrt{\left(\frac{(1+a)^2}{16}-\frac{1-\kappa}{2\tau}\right)\left(\frac{(1-a)^2}{16}-\frac{1-\kappa}{2\tau}\right)}.$$

Using that the condition in the Proposition implies $\frac{(1-a)^2}{16}-\frac{1-\kappa}{2\tau}>0$, both the term on the left-hand side and the term under the square root are positive. Squaring both sides of the inequality and rearranging results in

$$a^2>0 \quad \Leftrightarrow \quad p_W^{INT}>p_S^{INT}$$

which is what we wanted to show.

Corner equilibrium when the strong CP is prioritized: A corner equilibrium occurs when the prioritization of the stronger firm makes it so attractive that even the farthest consumer, the one located at 1 prefers buying its product. In this situation, the quality of the prioritized firm is K, whereas the quality of its competitor is 0. The strong firm can then extract all the surplus of the farthest customer and thus charges $p_S^C = v + \kappa - \tau(1-a)$. We must then show that

$$p_S^C < p_W^{INT} \quad \Leftrightarrow \quad v + \kappa - \tau(1-a) < v + 1 - \tau\left(\frac{3-a}{4} - \sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}}\right)$$

which after rearranging the terms rewrites as

$$\sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}} > \frac{3a-1}{4} - \frac{1-\kappa}{\tau}.$$

Notice that the left-hand side is always positive while the right-hand side is always negative as we assumed $a < 1/7$ and $K \leq 1$. Therefore $p_S^C < p_W^{INT}$ always holds.

Corner equilibrium when the weak CP is prioritized: A corner equilibrium occurs when the prioritization of the weaker firm makes it so attractive that even the farthest consumer, the one located at 0 prefers buying from it. In this situation, the quality of the prioritized firm is K, whereas the quality of its competitor is 0. The strong firm can then extract all the surplus of the farthest customer and thus charges $p_W^C = v + \kappa - \tau$. Notice that $p_W^C = v + \kappa - \tau < p_S^C = v + \kappa - \tau(1-a)$, and above we showed that $p_S^C < p_W^{INT}$, therefore $p_W^C < p_W^{INT}$ always holds.

Semi-corner equilibrium when the strong CP is prioritized: A semi-corner equilibrium occurs when the non-prioritized (weaker) firm provides 0 quality but still serves some consumers thanks to the large value of internet connection v. Such a situation arises whenever the indifferent consumer is located in the $[\kappa,1)$ interval, leading to the stronger firm providing quality κ/x. The location of the indifferent consumer x is given by

$$v + \frac{\kappa}{x} - p - \tau(x - a) = v + 0 - p - \tau(1 - x),$$

which leads to two potential solutions:

$$\frac{1+a}{4} - \sqrt{\frac{(1+a)^2}{16} + \frac{\kappa}{2\tau}} \quad \text{and} \quad x_S^{SC} = \frac{1+a}{4} + \sqrt{\frac{(1+a)^2}{16} + \frac{\kappa}{2\tau}}.$$

However, it is easy to see that the first one is negative, thus we can focus our attention to x_S^{SC}, leading to the price of

$$p_S^{SC} = v - \tau(1 - x_S^{SC}) = v - \tau \left(\frac{3-a}{4} - \sqrt{\frac{(1+a)^2}{16} + \frac{\kappa}{2\tau}} \right)$$

therefore

$$p_S^{SC} < p_W^{INT} \quad \Leftrightarrow \quad \sqrt{\frac{(1+a)^2}{16} + \frac{\kappa}{2\tau}} - \sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}} < \frac{1}{\tau}$$

Both sides of the inequality being positive, squaring them and rearranging leads to

$$L \equiv \frac{(1+a)^2}{16} + \frac{\kappa}{2\tau} - \frac{1}{4\tau} - \frac{1}{2\tau^2} < \sqrt{\frac{(1+a)^2}{16} + \frac{\kappa}{2\tau}} \sqrt{\frac{(1+a)^2}{16} - \frac{1-\kappa}{2\tau}}$$

This inequality is satisfied either if either (i) the term on left-hand side is non-positive, i.e. $L \leq 0$; or (ii) the square of both positive numbers also satisfy the inequality. (i) can be rewritten as

$$L \leq 0 \quad \Leftrightarrow \quad \kappa \leq \frac{4\tau - a^2\tau^2 - 2a\tau^2}{8\tau} + \frac{8 - \tau^2}{8\tau} \equiv \kappa',$$

whereas (ii) leads to

$$\kappa > \frac{4\tau - a^2\tau^2 - 2a\tau^2}{8\tau} + \frac{4}{8\tau} \equiv \kappa''.$$

For $\tau \leq 2$ we have $\kappa'' \leq \kappa'$ thus (i) or (ii) is satisfied for any κ, therefore $p_S^{SC} < p_W^{INT}$.

For $\tau > 2$, it is easy to check that

$$\frac{1+\tau-a\tau}{2\tau} > \kappa''$$

thus the condition of the Proposition, $\kappa \geq \frac{1+\tau-a\tau}{2\tau}$, guarantees that (ii) is always satisfied, leading to $p_S^{SC} < p_W^{INT}$, which concludes this part of the proof.

Semi-corner equilibrium when the weak CP is prioritized: Finally, we investigate the equilibria leading to a situation where the non-prioritized (stronger) firm provides 0 quality while still serving some consumers due to the large value of internet connection v. Such a situation arises whenever the indifferent consumer (at position x) is located in the $[0, 1-\kappa)$ interval. In a semi-corner equilibrium the weaker firm provides quality $\kappa/(1-x)$ to the $1-x$ consumers closest to it.

We have to distinguish two cases depending on the location of the indifferent consumer. Firstly, if it is located to the right of CP1, i.e. $x \in (a, 1-\kappa)$, then x is given by

$$v + 0 - p - \tau(x-a) = v + \frac{\kappa}{1-x} - p - \tau(1-x),$$

which leads to two potential solutions:

$$x_1 = \frac{3+a}{4} - \sqrt{\frac{(1-a)^2}{16} + \frac{\kappa}{2\tau}} \quad \text{and} \quad x_2 = \frac{3+a}{4} + \sqrt{\frac{(1-a)^2}{16} + \frac{\kappa}{2\tau}}.$$

However, it is easy to show that $x_2 > 1$, thus we can focus our attention to x_1. Notice that one of the conditions for x_1 being a semi-corner equilibrium, $x_1 < 1-\kappa$, rewrites as

$$\kappa - \frac{1-a}{4} < \sqrt{\frac{(1-a)^2}{16} + \frac{\kappa}{2\tau}}.$$

The condition of the Proposition, $\kappa \geq \frac{1+\tau-a\tau}{2\tau}$, implies that $\kappa > \frac{1-a}{2}$ thus both sides of the inequality are positive. Squaring and rearranging reveals that $\kappa < \frac{1+\tau-a\tau}{2\tau}$ is a necessary condition for the existence of such an equilibrium, which is ruled out by the condition of the Proposition.

Secondly, if the indifferent consumer is located to the left of CP1, i.e., $x \in (0, a]$, then its location is given by

$$v + 0 - p - \tau(a - x) = v + \frac{\kappa}{1 - x} - p - \tau(1 - x),$$

which leads to

$$x_W^{SC3} = 1 - \frac{\kappa}{\tau(1-a)} \quad \text{and} \quad p_W^{SC3} = v - \frac{\kappa}{1-a} + \tau(1-a).$$

Notice that one of the necessary conditions for the existence of such a semi-corner solution is

$$x_W^{SC3} < a \quad \Leftrightarrow \quad \tau(1-a)^2 < \kappa,$$

therefore $\tau(1-a)^2 < 1$ is also necessary, otherwise it would require $\kappa > 1$.

Next, notice that the price p_W^{SC3} is decreasing in κ whereas the price p_W^{INT} is increasing in κ. We will prove $p_W^{SC3} < p_W^{INT}$ by showing that it is satisfied even at the lower bound of possible values of κ. To see this, replacing $\kappa = \tau(1-a)^2$ into the prices we have

$$p_W^{SC3} < p_W^{INT} \quad \Leftrightarrow \quad \frac{3-a}{4} - \frac{1}{\tau} < \sqrt{\frac{(1+a)^2}{16} + \frac{(1-a)^2}{16} - \frac{1}{2\tau}}.$$

It is sufficient to show that the expression on the left-hand side is negative which is equivalent to $\tau < \frac{4}{3-a}$. Using the existence condition of $\tau(1-a)^2 < 1$ derived above, it is sufficient to show that $\frac{1}{(1-a)^2} < \frac{4}{3-a}$. Solving this quadratic inequality reveals that this is satisfied for $a < \frac{7-\sqrt{33}}{8} \approx 0.157$ which in turn follows from the assumption $a < 1/7$. This concludes the proof of Proposition 1. ■

Proof of Lemma 3

Aggregate consumer surplus under net neutrality writes as

$$CS^{NN} = v + \kappa - p^{NN} - \int_0^a \tau(a-t)\mathrm{d}t - \int_a^x \tau(t-a)\mathrm{d}t - \int_x^1 \tau(1-t)\mathrm{d}t = \tau\left(\frac{1}{4} - \frac{3}{4}a^2\right).$$

Under the optimal zero-rating program, consumer surplus equals

$$CS^{ZR} = v + \kappa - \int_0^a p + d + \tau(a-t)\mathrm{d}t - \int_a^x p + d + \tau(t-a)\mathrm{d}t - \int_x^1 p + \tau(1-t)\mathrm{d}t$$

which simplifies to

$$CS^{ZR} = \tau\left(\frac{1}{4} - \frac{15}{16}a^2\right) = CS^{NN} - \tau\frac{3}{16}a^2.$$

The consumer surplus under paid prioritization when the weak firm has priority is the following:

$$CS^P = v - p^{NN} + \int_0^a q_1 - \tau(a-t)\mathrm{d}t + \int_a^x q_1 - \tau(t-a)\mathrm{d}t + \int_x^1 q_2 - \tau(1-t)\mathrm{d}t.$$

The comparison between CS^P and CS^{NN} gives the condition of Lemma 3. The comparison between CS^P and CS^{ZR} gives the following:

$$CS^{ZR} \geq CS^P \text{ if } \kappa \in [\frac{8 - 6a\tau + 3a^2\tau}{8}, \frac{8 - 2a\tau - a^2\tau}{8}]$$

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